Geometry
For
Elementary School Teachers

A Professional Development Training Program
To Implement the
2001 Virginia Standards of Learning

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Notice to Reader

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Introduction

The updated *Geometry for Elementary School Teachers* is a professional development training program designed to assist teachers in implementing the 2001 Virginia Standards of Learning for mathematics. This professional development program provides a sample of meaningful and engaging activities correlated to the geometry strand of the K-5 Mathematics Standards of Learning.

The purpose of the professional development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching the geometry strand of the K-5 Mathematics Standards of Learning. Teachers will learn about the van Hiele model for the development of geometric thought and how this can be used to guide instruction and classroom assessment. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and activities that address many of the dimensions of geometry including spatial relationships, properties of geometric figures, constructions, geometric modeling, geometric transformations, coordinate geometry, the geometry of measurement, informal geometric reasoning, and geometric connections to the physical world. Teachers will explore two- and three-dimensional figures, paper folding and origami, tessellations and geometric designs, and the use of other manipulatives to develop geometric understanding. Through these activities, it is anticipated that teachers will develop new techniques that assist in increasing student achievement in their classrooms.

Designed to be presented by teacher trainers, this professional development program includes directions for the trainer, as well as the activity sheets for participants. An addendum to the module includes video segments of the van Hiele levels. These video segments portray students engaged in assessment tasks and can be used to discuss the students' level of development of geometric thought. Directions for the assessment tasks are also included.

Trainers should adapt the materials to best fit the needs of their audience; adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. All materials in this document may be duplicated and distributed as desired for use in Virginia.

The training program is organized into five three-hour modules that may be offered by school divisions for recertification points or for a one-credit graduate course, when university credit can be arranged.

GLOSSARY

Acute Angle	An angle with a measure greater than 0 degrees but less than 90 degrees.
Acute Triangle	A triangle with three acute angles (or no angle measuring 90 degrees or greater).
Adjacent Sides	Two sides of a polygon with a common vertex.
Angle	Two rays that share an endpoint.
Arc	Part of a circle.
Area	The amount of surface in a region or enclosed within a boundary. Area is measured in square units such as square feet or square centimeters.
Attribute	A characteristic possessed by an object. Characteristics include shape, color, size, length, weight, capacity, area, etc.
Base of a Solid	A plane figure. If the solid is a cylinder or prism, there are two bases that are parallel and congruent.
Centimeter	A metric unit of length equal to one-hundredth of one meter.
Circle	A closed curve with all points in one plane and equidistant from a fixed point (the center).
Circumference	The length of the boundary of a circular region. The circumference can be computed by multiplying the diameter by *pi* (π), a number a little more than 3.14.
Concentric Circles	Two or more circles that have the same center and different radii.
Cone	A three-dimensional figure with a circular base joined to a vertex by a curved surface.
Congruent	Having exactly the same size and shape. Congruent polygons have their corresponding angles congruent and corresponding sides congruent.
Coordinate System	A reference system for locating and graphing points. In two dimensions, a coordinate system usually consists of a horizontal axis and a vertical axis, which intersect at the origin. Each point in the plane is located by its horizontal distance and vertical distance from the origin. These distances, or coordinates, form an ordered pair of numbers.
Cube	A solid figure in which every face is a square and every edge is the same length.

Cubic Foot	The volume of a cube that is one foot wide, one foot high, and one foot long.
Cubic Unit	A unit of measure that has a length of one unit, a width of one unit, and a height of one unit used to measure volume. Examples are cubic inches, cubic centimeters, etc.
Cylinder	A solid figure formed by two congruent parallel circles joined by a curved surface.
Decagon	A polygon with ten sides. A regular decagon has ten congruent sides and ten congruent angles.
Diagonal	A line segment that joins two non-adjacent vertices of a polygon or polyhedron.
Diameter	A line segment passing through the center of a circle or sphere and connecting two points on the circle or sphere.
Diamond	See *Rhombus.*
Dimension	The number of coordinates used to express a position.
Dodecagon	A polygon with twelve sides. A regular dodecagon has twelve congruent sides and twelve congruent angles.
Dodecahedron	A polyhedron with twelve faces. All faces of a regular dodecahedron are congruent, regular pentagons.
Edge	A line segment where two faces of a three-dimensional figure meet.
Endpoint	The point(s) at the end of a ray or line segment.
Equilateral Triangle	A triangle with three congruent sides. Each angle measures 60 degrees.
Face	A plane figure that serves as one side of a solid figure.
Flip	(See Reflection)
Geometry	The branch of mathematics that deals with the position, size, and shape of figures.
Grid	A network of horizontal and vertical lines that intersect to form squares or rectangles.
Hemisphere	Half of a sphere, formed by making a plane cut through the center of a sphere.

Heptagon	A polygon with seven sides. A regular heptagon has seven congruent sides and seven congruent angles.
Hexagon	A polygon with six sides. A regular hexagon has six congruent sides and six congruent angles.
Hexahedron	A polyhedron with six faces. A regular hexahedron is a cube.
Hypotenuse	The side opposite the right angle of a right triangle. The hypotenuse is the longest side of a right triangle.
Icosahedron	A polyhedron with twenty faces. All faces of a regular icosahedron are congruent, equilateral triangles.
Isosceles Triangle	A triangle with at least two congruent sides and two congruent angles. (An equilateral triangle is a special case of an isosceles triangle.)
Kite	A convex quadrilateral with two distinct pairs of adjacent, congruent sides.
Line	A set of points that form a straight path extending infinitely in two directions. Lines are often called “straight lines” to distinguish them from curves, which are often called “curved lines.” Part of a line with two endpoints is called a “line segment”.
Line of Symmetry	A line dividing a two-dimensional figure into two parts that are mirror images of each other.
Line Segment	A part of a line. A line segment has two endpoints and a finite length.
Network	A diagram consisting of arcs (branches) connecting points or nodes (junctions). A network may represent a real-world situation, such as a road system or an electronic circuit. Sometimes the nodes are called vertices.
Node	A point in a network at the end of an arc or at the junction of two or more arcs.
Nonagon	A polygon with nine sides. A regular nonagon has nine congruent sides and nine congruent angles.
Obtuse Angle	An angle that is greater than 90 degrees but less than 180 degrees; that is, between a right angle and a straight line.
Obtuse Triangle	A triangle that has one obtuse angle.
Octagon	A polygon with eight sides. A regular octagon has eight congruent sides and eight congruent angles.

Octahedron	A polyhedron with eight faces. All faces of a regular octahedron are congruent, equilateral triangles.
Opposite Angles	In a quadrilateral, angles that do not have a common side; non-adjacent angles.
Parallel Lines	Lines lying in the same plane that are always the same distance apart.
Parallelogram	A quadrilateral with both pairs of opposite sides parallel. Opposite angles are congruent.
Pentagon	A polygon with five sides. A regular pentagon has five congruent sides and five congruent angles.
Perimeter	The distance around a figure.
Perpendicular	At right angles.
pi (π)	The ratio of the circumference of a circle to its diameter. This ratio is the same for every circle. Its value, which is found by dividing the circumference by the diameter, is a little more than 3.14.
Pie graph	A circle marked into sectors. Each sector shows the fraction represented by one category of data. Pie graphs are also called circle graphs.
Plane	A flat surface extending infinitely in all directions.
Plane Figure	In geometry, a closed two-dimensional figure that lies entirely in one plane. (Polygons and circles are examples of plane figures. An arc is not a plane figure because it is not closed.)
Point	The smallest geometric unit. A position in space, often represented by a dot.
Polygon	A simple, closed, plane figure bounded by straight sides.
Polyhedron	A solid figure bounded by flat faces.
Prism	A polyhedron with at least one pair of opposite faces that are parallel and congruent. Corresponding edges of these faces are joined by rectangles or parallelograms.
Pyramid	A polyhedron with any polygon for its base. The other faces are triangles that meet at a point or vertex.
Quadrilateral	A polygon with four sides.

Ray — A set of points that form a straight path extending infinitely in one direction. A ray has one endpoint.

Rectangle — A parallelogram with four right angles. Opposite sides are congruent and parallel.

(Right) Rectangular Prism — A solid figure in which all six faces are rectangles with three pairs of parallel, congruent, opposite faces.

Reflection — A transformation of a geometric figure that results in a mirror image of the original.

Regular Polygon — A polygon that has equal sides and equal angles.

Regular Polyhedron — A polyhedron with congruent faces that are regular polygons.

Rhombus — A parallelogram with four congruent sides. Opposite angles are congruent.

Right Angle — An angle that is one-fourth of a full turn. A right angle measures 90 degrees.

Right Triangle — A triangle that has one right angle.

Scalene Triangle — A triangle with no sides congruent.

Semicircle — One-half of a circle, also called a semi-circle.

Similar — Figures that have the same shape but not necessarily the same size. Similar polygons have corresponding angles congruent and corresponding sides in proportion. Congruent is a special case of similar where the ratio of the corresponding sides is 1-1.

Slide — (See Translation)

Solid Figure — A closed, three-dimensional figure.

Sphere — A three-dimensional figure formed by a set of points that are all the same distance from a fixed point called the center.

Square — A rectangle with congruent sides.

Square Unit — A unit of measure that has a length of one unit and a width of one unit used to measure area. Examples are square inches, square centimeters, acres, etc.

Surface — Part or all of the boundary of a solid. A surface may be flat or curved. (For example, a cone has one flat surface and one curved surface).

Symmetry

a. If a figure can be folded along a line so that the two halves match exactly, then the figure has line symmetry.

b. If a figure can be turned less than 360 degrees about a point and fit exactly on itself, then a figure has turn (or rotational) symmetry.

Tessellation

An arrangement of plane figures (usually congruent figures) to cover a surface without overlapping or leaving any gaps.

Tetrahedron

A polyhedron with four triangular faces. A tetrahedron is a triangular pyramid.

Three-Dimensional

Relating to objects that have length, width, and depth. Solid figures such as polyhedra, cones, and spheres are three-dimensional.

Transformation

Moving a geometric figure from one position to another, according to a rule. Examples of transformations are reflection, rotation, and translation.

Translation

A transformation in which a geometric figure is formed by moving every point on a figure the same distance in the same direction.

Trapezoid

A quadrilateral with exactly one pair of parallel sides.

Triangle

A polygon with three sides.

Triangular Prism

A prism in which the bases are triangles.

Two-Dimensional

Relating to figures that have length and width but not depth. Figures such as polygons and circles are two-dimensional.

Vertex

a. A point at which two line segments, lines, or rays meet to form an angle.
b. A point on a polyhedron where three or more faces intersect.
The plural of vertex is vertices.

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Geometry Standards of Learning
Kindergarten through Fifth Grade

* indicates a related Standard of Learning from the Measurement Strand

K.11 The student will identify, describe, and draw two-dimensional (plane) geometric figures (circle, triangle, square, and rectangle).

K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.

K.13 The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).

1.15 The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside,* and *next to).*

1.16 The student will draw, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and square corners.

1.17 The student will identify and describe objects in his/her environment that depict plane geometric figures (triangle, rectangle, square, and circle).

2.20 The student will identify, describe, and sort three-dimensional (solid) concrete figures, including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners.

2.21 The student will identify and create figures, symmetric along a line, using various concrete materials.

2.22 The student will compare and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle/rectangular solid).

3.18 The student will analyze two-dimensional (plane) and three-dimensional (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular solid [prism], square pyramid, sphere, cone, and cylinder) and identify relevant properties, including the number of corners, square corners, edges, and the number and shape of faces, using concrete models.

3.19 The student will identify and draw representations of line segments and angles, using a ruler or straightedge.

3.20 The student, given appropriate drawings or models, will identify and describe congruent and symmetrical, two-dimensional (plane) figures, using tracing procedures.

4.13* The student will
- a) identify and describe situations representing the use of perimeter and area; and
- b) use measuring devices to find perimeter in both standard and nonstandard units of measure. (*Measurement strand)

4.14 The student will investigate and describe the relationships between and among points, lines, line segments, and rays.

4.15 The student will
- a) identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and
- b) describe the path of shortest distance between two points on a flat surface.

4.16 The student will identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity.

4.17 The student will
- a) analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]);
- b) identify congruent and noncongruent shapes; and
- c) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide) and rotation (turn), using mirrors, paper folding, and tracing.

4.18 The student will identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane.

5.8* The student will describe and determine the perimeter of a polygon and the area of a square, rectangle, and right triangle, given the appropriate measures. (*Measurement strand)

5.10* The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation. (*Measurement strand)

5.13* The student will measure and draw right, acute, and obtuse angles and triangles, using appropriate tools. (*Measurement strand)

5.14 The student will classify angles and triangles as right, acute, or obtuse.

5.15 The student, using two-dimensional (plane) figures (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will

a) recognize, identify, describe, and analyze their properties in order to develop definitions of these figures;
b) identify and explore congruent, noncongruent, and similar figures;
c) investigate and describe the results of combining and subdividing shapes;
d) identify and describe a line of symmetry; and
e) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).

5.16 The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism).

Elementary Geometry
Session 1

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
van Hiele Theory of Geometric Thought	Lecture – van Hiele Levels of Geometric Thought Triangle Sorts	3	K.11, K.12, K.13, 1.16 3.18, 4.17, 5.15	Explanation Sheets -van Hiele levels, -Additional Points, Triangle Sorting Pieces, Pages 1-4 Sample Student Sorts 1-6	Paper triangles
Quadrilaterals and Their Properties	Quadrilateral Sort	19	K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15	Quadrilateral Sorting Pieces	Paper quadrilaterals
	What's My Rule?	22	K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15	What's My Rule?	Paper quadrilaterals
	Quadrilateral Properties Laboratory	24	K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15	Types of Quadrilaterals	Geo-strips, D-stix, or miniature marshmallows and toothpicks; square corner
	Quadrilateral Sorting Laboratory	28	K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15	Quadrilateral Sorting Laboratory, Quadrilateral Table, Quadrilateral Family Tree	Paper quadrilaterals

Topic: The van Hiele Theory of Geometric Thought

Description: The van Hiele theory of geometric thought describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. In this first session, the participants will explore the van Hiele levels of geometric thought by doing triangle sorts and comparing their sorts to those performed by elementary students. The sorting task is appropriate for all ages and levels of students. It can serve as an activity to help students advance their level of understanding as well as an assessment tool that can inform the teacher at what van Hiele level the student is thinking.

Related SOL: K.11, K.12, K.13, 1.16, 3.18, 4.17, 5.15

Activity: The van Hiele Levels of Geometric Thought

Format: Large Group Lecture and Small Group Activity

Objectives: Participants will be able to describe the developmental sequence of geometric thinking according to the van Hiele theory of geometric thought and activities suitable for each level. In addition, participants will be able to assess the van Hiele levels of their students.

Related SOL: K.11, K.12, K.13, 1.16, 3.18, 4.17, 5.15

Materials: Paper triangles, cut out and placed in a plastic baggy or manila envelope (see Activity Sheet for Triangle Sorting Pieces.) You will need at least one set of triangles for every three participants.

Time Required: Approximately 1 hour

Background: To Trainer (for lecture):
After observing their own students, Dutch teachers P.M. van Hiele and Dina van Hiele-Geldof described learning as a discontinuous process with jumps that suggest "levels." They identified five sequential levels of geometric understanding or thought:

1) Visualization
2) Analysis
3) Abstraction
4) Deduction
5) Rigor

Clements and Battista (1992) proposed the existence of a Level 0 that they called Pre-recognition.

In Kindergarten through grade two most students will be at Level 1. By grade three, students should be transitioning to Level 2. If the content in the Virginia Standards of Learning is mastered, students should attain Level 3 by the end of sixth grade. Level 4 is usually attained by students who can prove theorems using deductive techniques. One problem is that most current textbooks provide activities requiring only Level 1 thinking up through sixth grade and teachers must provide different types of tasks to facilitate the development of the higher levels of thought.

Directions:

I. Participants should study the The van Hiele Levels Explanation Sheet.

2. Turn to the Additional Points Explanation Sheet. Note for Point 1 that the levels are hierarchical. Students cannot be expected to write a geometric proof successfully unless they have progressed through each level of thought in turn. At Point 2, college students and even some teachers have been found who are at Level 1, while there are middle schoolers at Level 3 and above. (If the content in the SOL is mastered, students should attain Level 3 by the end of 6^{th} grade.) As an example of an experience that can impede progress (Point 3), think of the illustration of the teacher who knew that the relationship between squares and rectangles was a difficult one for her fourth graders so she had them memorize, “Every square is a rectangle, but not every rectangle is a square.” When tested a few weeks later, half the students remembered that a square is a type of rectangle, while the other half thought that a rectangle was a type of square. It was almost impossible for these students to learn the true relationship between squares and rectangles because every time they heard the words square and rectangle together, they insisted on relying on their memorized sentence rather than on the properties of the two types of figures.

3. Continue on to Properties of Levels. As an example of separation, consider the meaning of the word “square.” When a teacher thinking at Level 3 or above says “square”, the word conveys the properties and relationships of a square: having four congruent sides; having four congruent angles; having perpendicular diagonals; and being a type of polygon, quadrilateral, parallelogram, and rectangle. To a student thinking at Level 1, the word “square” will only evoke an image of something that looks like a square such as a CD case or first base. The same word is being used, but it has an entirely different meaning to the teacher and the student. The teacher must keep in mind what the meaning of the word or symbol is to the student and how the student thinks about it. For Attainment, it is important to note that there are five phases of learning that lead to understanding at the next higher level.

4. Divide participants into small groups. Distribute the sets of cutout triangles, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them triangles. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they

are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way to sort the flag parts would be to put all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. If needed, you can demonstrate a triangle sort using pieces cut from the Triangle Sorting Pieces Activity Sheet. Have participants sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

5. Ask the participants to describe their sorts. Expect answers like "acute, right, and obtuse triangles" or "scalene, isosceles, and equilateral". Have them compare their sorts with those of other groups.

6. Ask them how they think their students would sort these figures. Refer to Sample Student Sort Sheets and ask the participants to conjecture the criteria used for sorting and the van Hiele level of the sorter. Sample Student Sort 1 is a low Level 1 sort where the student is sorting strictly by size and may not even know that the figures are triangles. Sample Student Sort 2 is another Level 1 sort. Here the student thinks that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Sample Student Sort 3 is another Level 1 sort. This student also believes that triangles must have at least two sides the same length or possibly that triangles must be symmetric. Additionally, this student recognized the figures with right angles or "corners" as a separate category. The Sample Student Sort 4 is at least a Level 2 or 3 sort in which the sorter focuses on the lengths of the sides, a criterion that separates the figures into categories that overlap. The student has actually sorted into groups with no sides the same length, two sides the same length, and all sides the same length. It is unclear whether the student knows that equilateral triangles are a type of isosceles triangle. The Sample Student Sort 5 focuses on parts of the figures and so is a Level 2 sort, but the student does not have the vocabulary to adequately describe the figures. The Sample Student Sort 6 is similar to the Sample Student Sort 4, but the word "Perfect" is incorrect and indicates that the student may be thinking more of the figure as a whole rather than of the individual parts. This sort is probably Level 2.

Explanation Sheet: The van Hiele Levels

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of the figure. A student should recognize and name figures, and distinguish a given figure from others that look somewhat the same. “I know it's a rectangle because it looks like a door and I know that the door is a rectangle."

Level 2: Analysis. Properties are perceived, but are isolated and unrelated. A student should recognize and name properties of geometric figures. “I know it's a rectangle because it is closed, it has four sides and four right angles, opposite sides are parallel, opposite sides are congruent, diagonals bisect each other, adjacent sides are perpendicular,...”

Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. “I know it’s a rectangle because it's a parallelogram with right angles."

Level 4: Deduction. The student can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. A student should be able to supply reasons for steps in a proof.

Level 5: Rigor. The standards of rigor and abstraction represented by modern geometries characterize level 5. Symbols without referents can be manipulated according to the laws of formal logic. A student should understand the role and necessity of indirect proof and proof by contrapositive.

Explanation Sheet: **Additional Points**

1. The learner cannot achieve one level without passing through the previous levels.
2. Progress from one level to another is more dependent on educational experience than on age or maturation.
3. Certain types of experiences can facilitate or impede progress within a level or to a higher level.

Properties of Levels

Adjacency: What was intrinsic in the preceding level is extrinsic in the current level.

Distinction: Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Separation: Two individuals reasoning at different levels cannot understand one another.

Attainment: The learning process leading to complete understanding at the next higher level has five phases: inquiry/information, directed orientation, explication, free orientation and integration.

Phases of Learning

Inquiry/Information: Gets acquainted with the working domain (e.g., examines examples and non-examples)

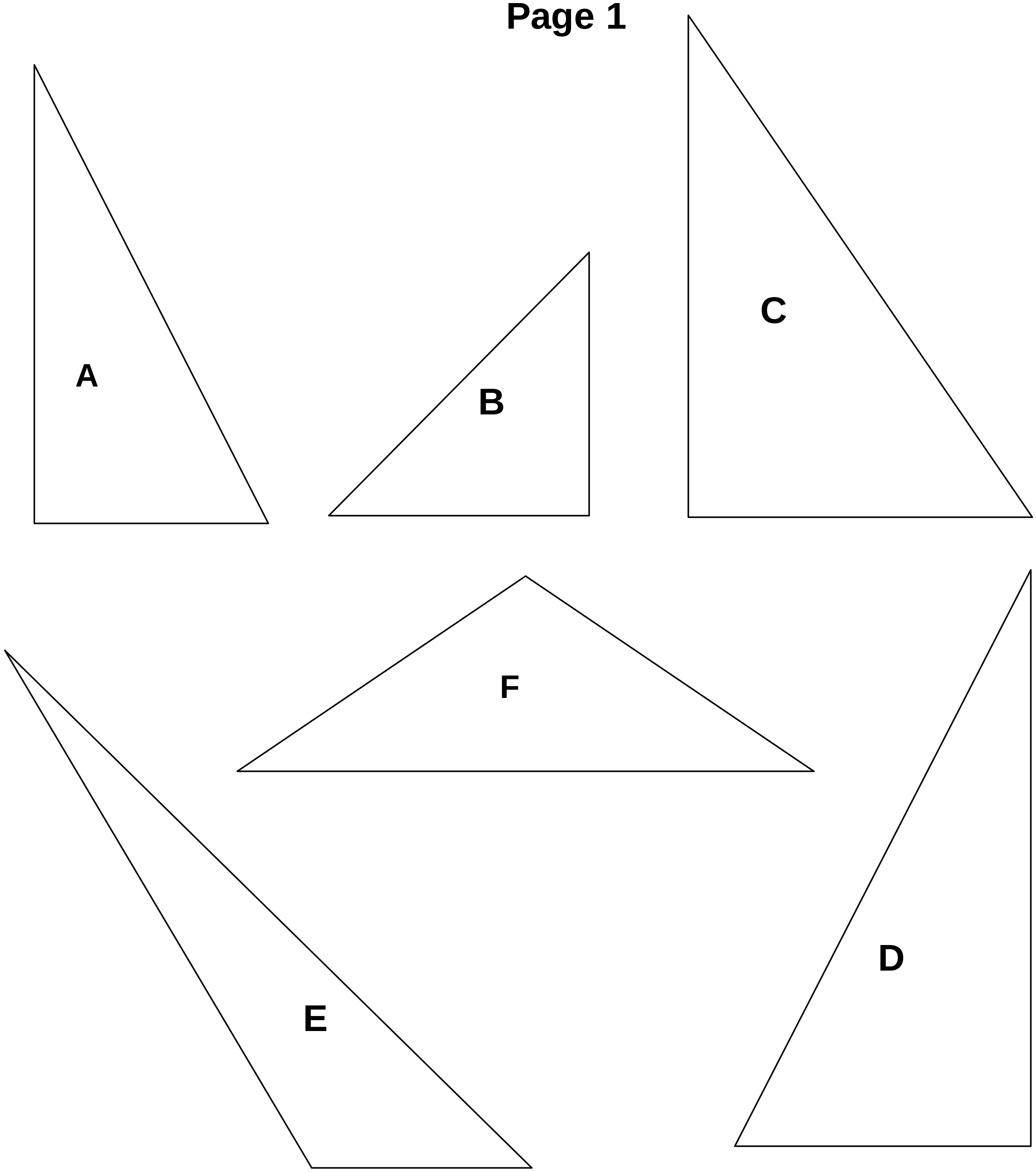
Guided orientation: Does tasks involving different relations of the network that is to be formed (e.g., folding, measuring, looking for symmetry)

Explication: Becomes conscious of the relations, tries to express them in words, and learns technical language which accompanies the subject matter (e.g., expresses ideas about properties of figures)

Free orientation: Learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g., knowing properties of one kind of figure, investigating these properties for a new figure, such as kites)

Integration: Summarizes all that has been learned about the subject, then reflects on actions, and obtains an overview of the newly formed network of relations now available (e.g., properties of a figure are summarized)

Triangle Sorting Pieces
Page 1



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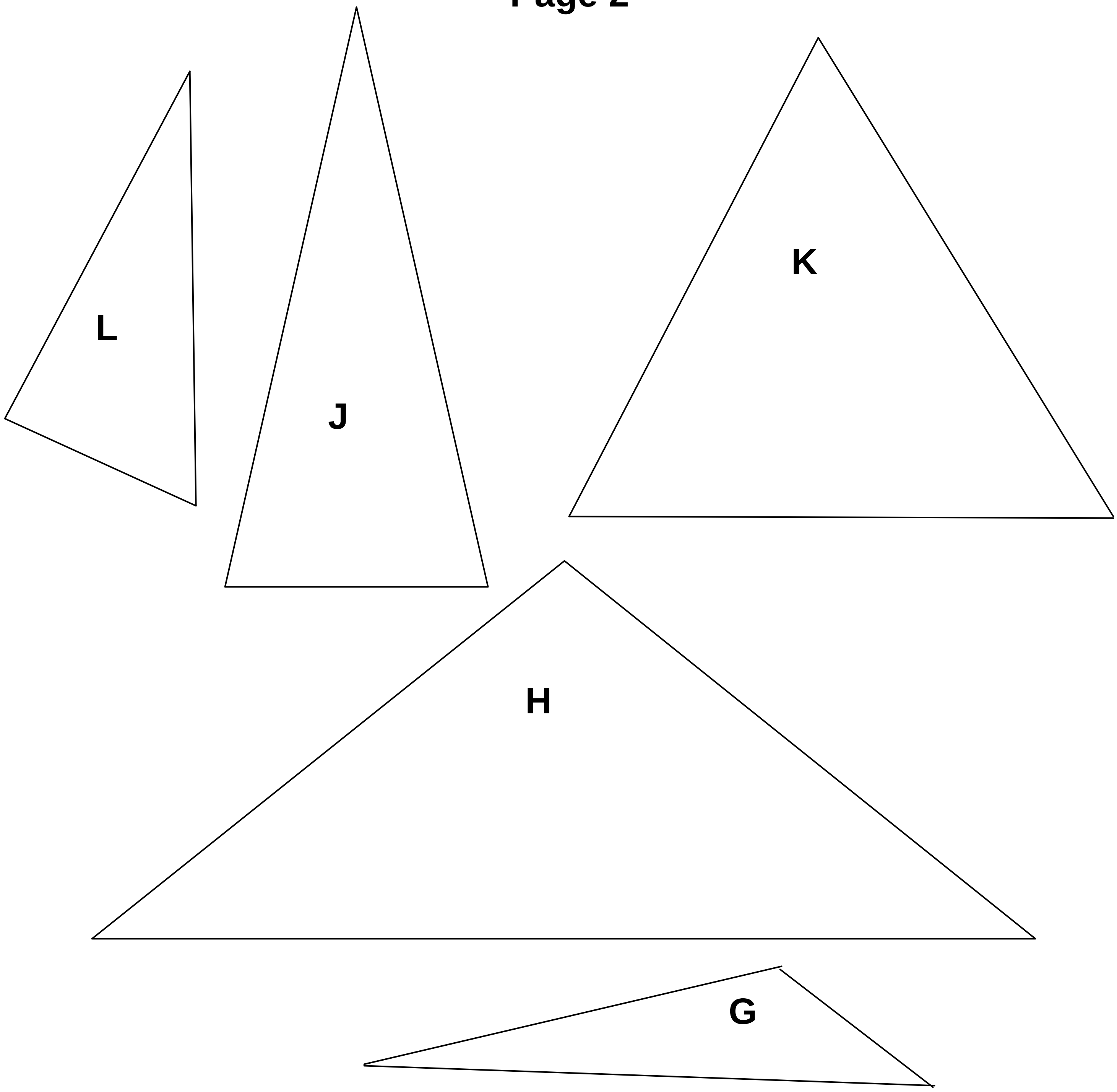

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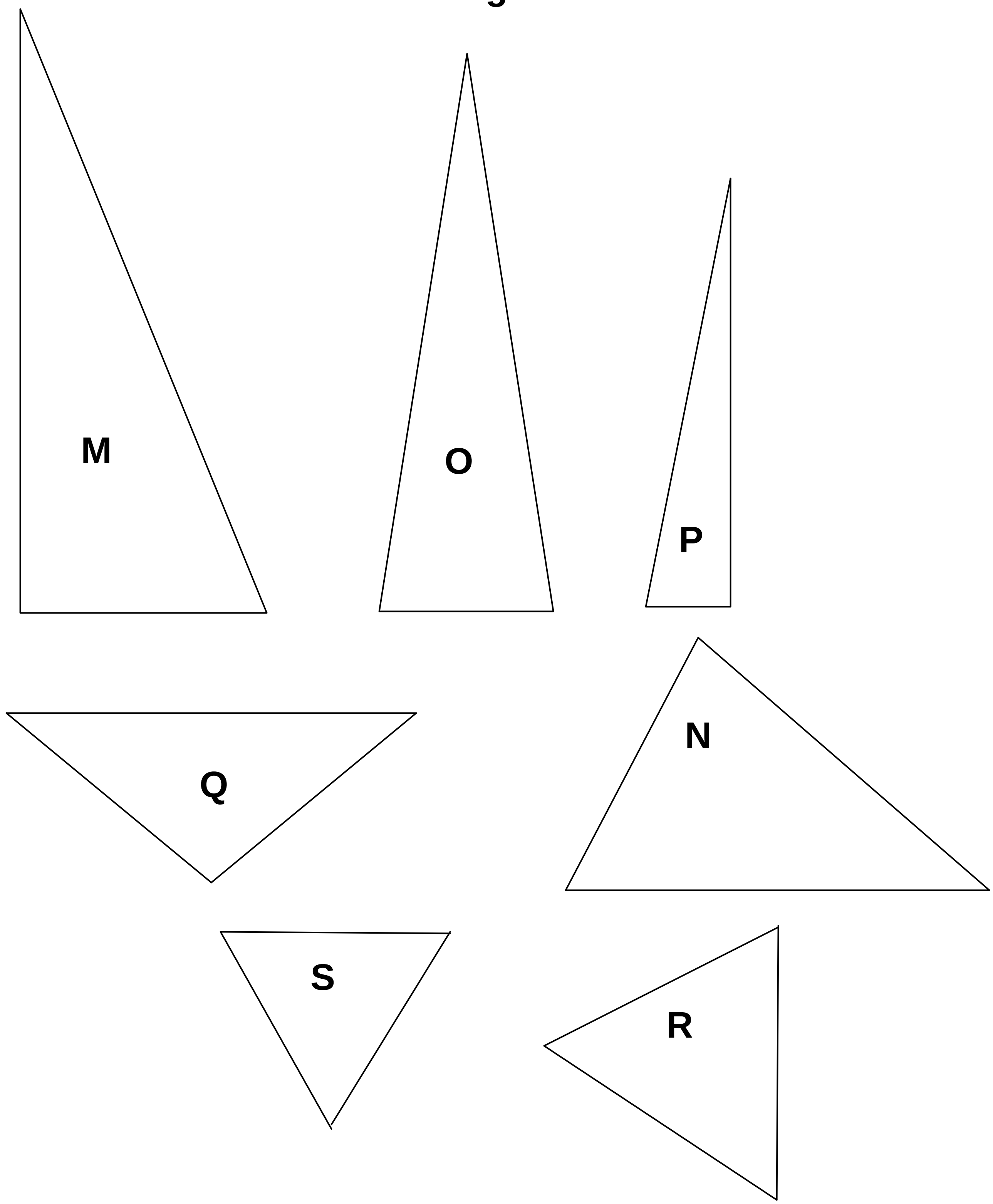
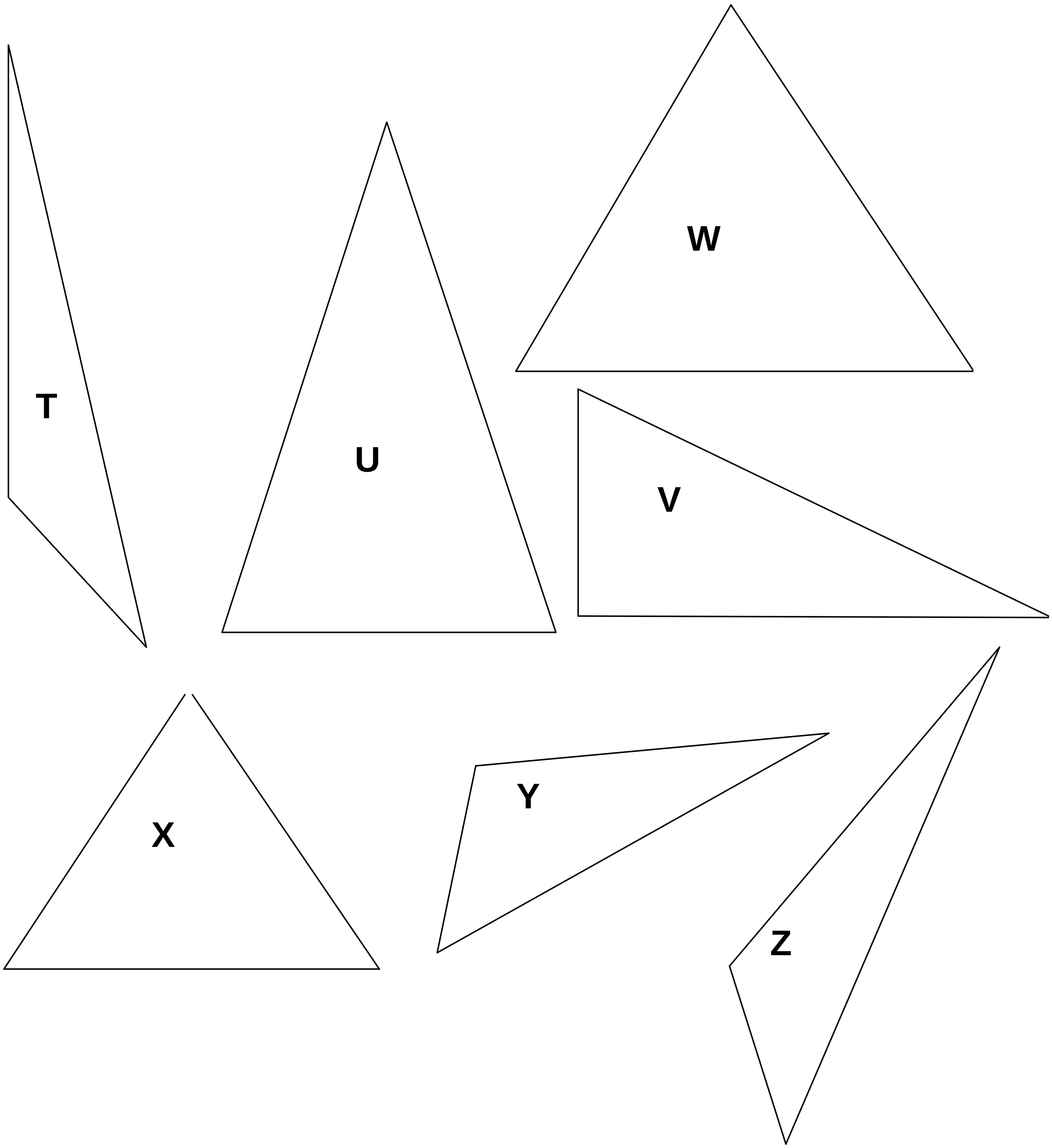
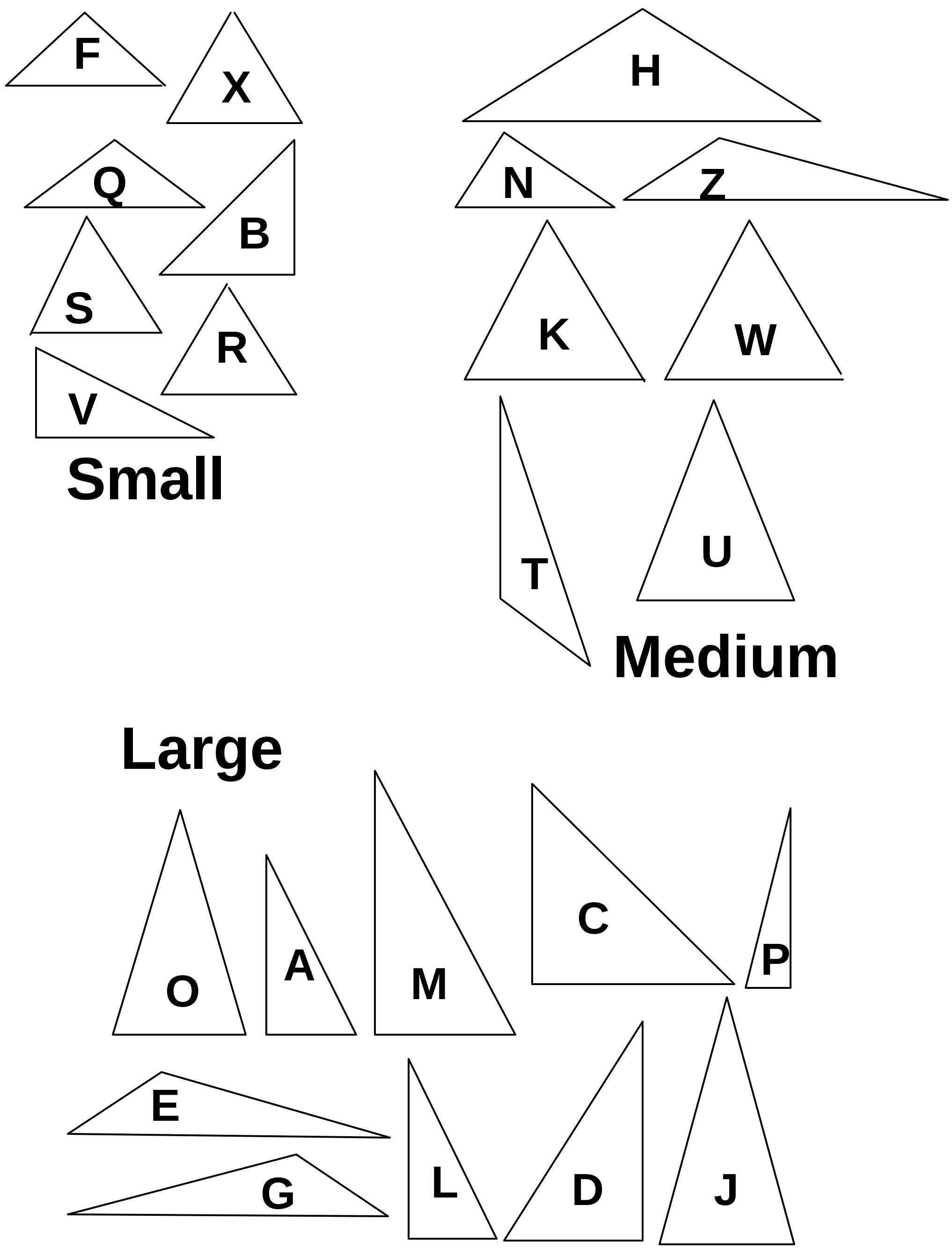


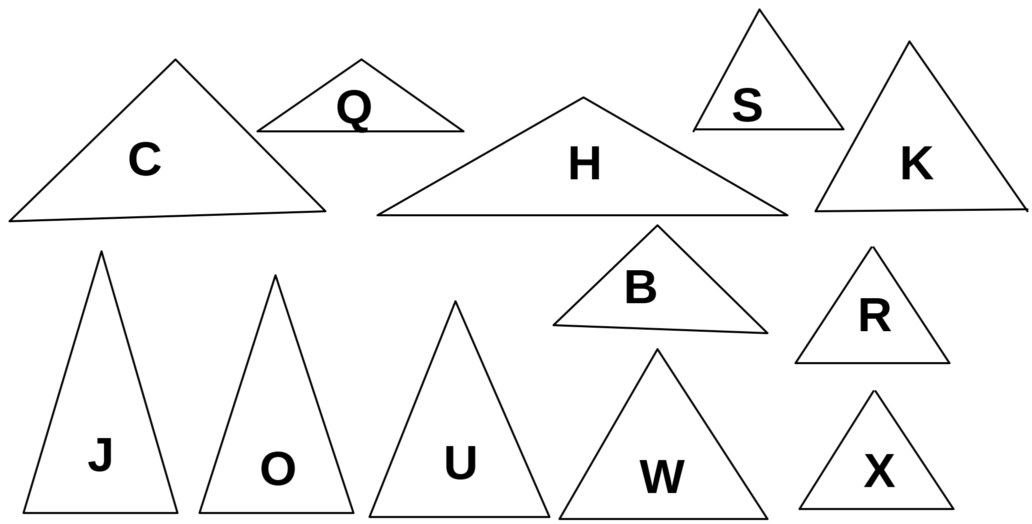
Triangle Sorting Pieces
Page 4



Sample Student Sort 1

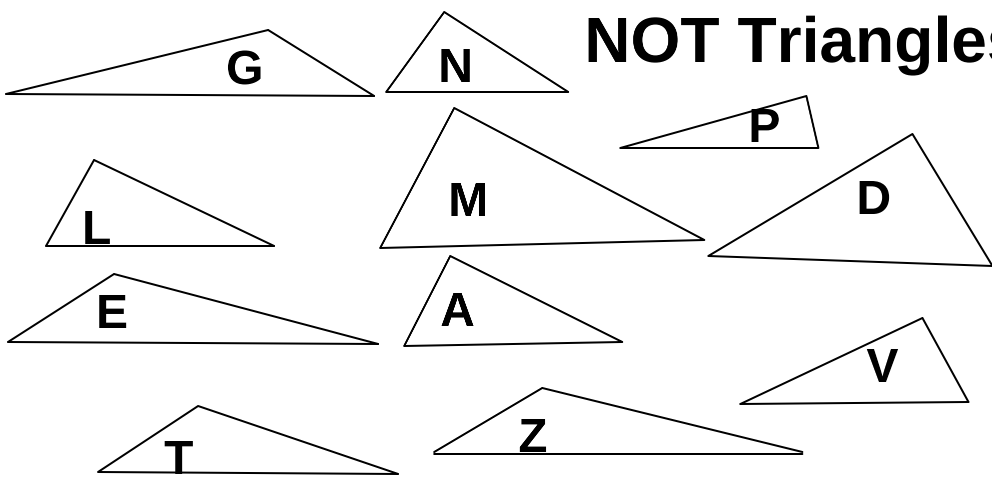


Sample Student Sort 2

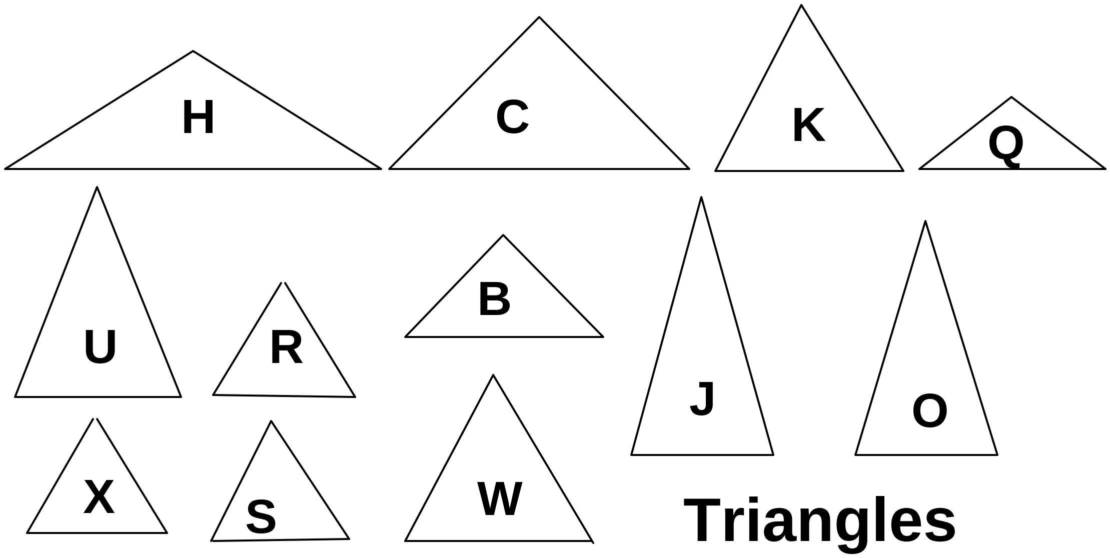


Triangles

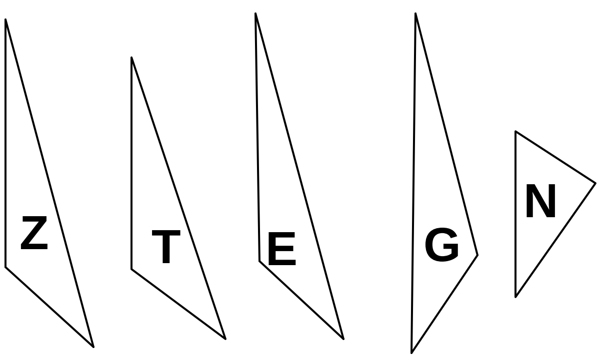
NOT Triangles



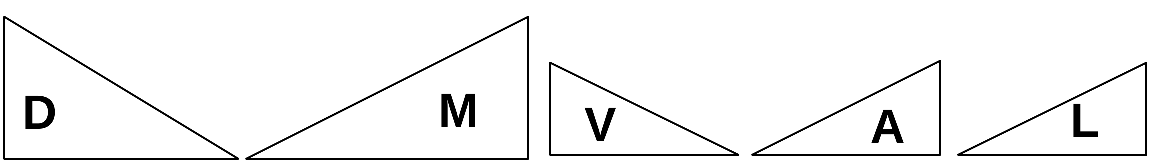
Sample Student Sort 3



Triangles

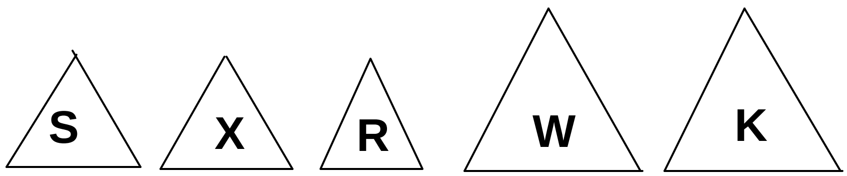


Look alike...
N is smaller...
NOT triangles

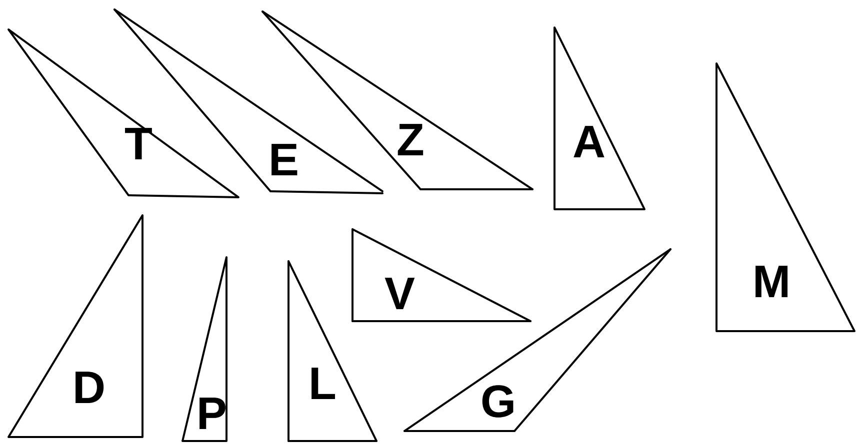


Look like ramps... NOT triangles

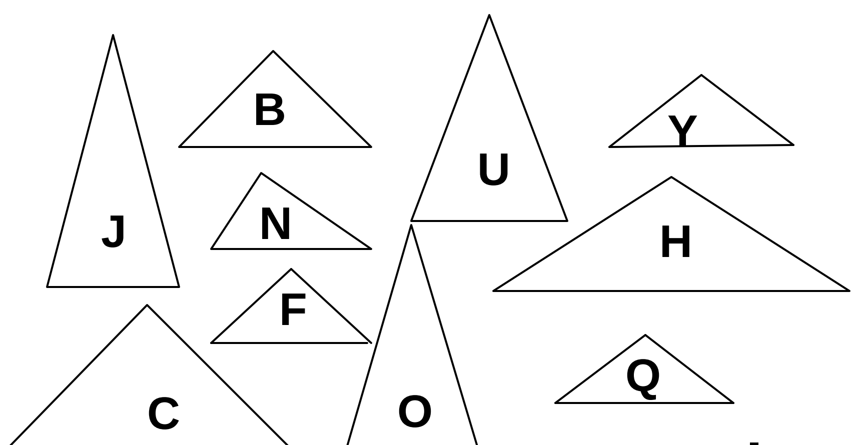
Sample Student Sort 4



Equilateral

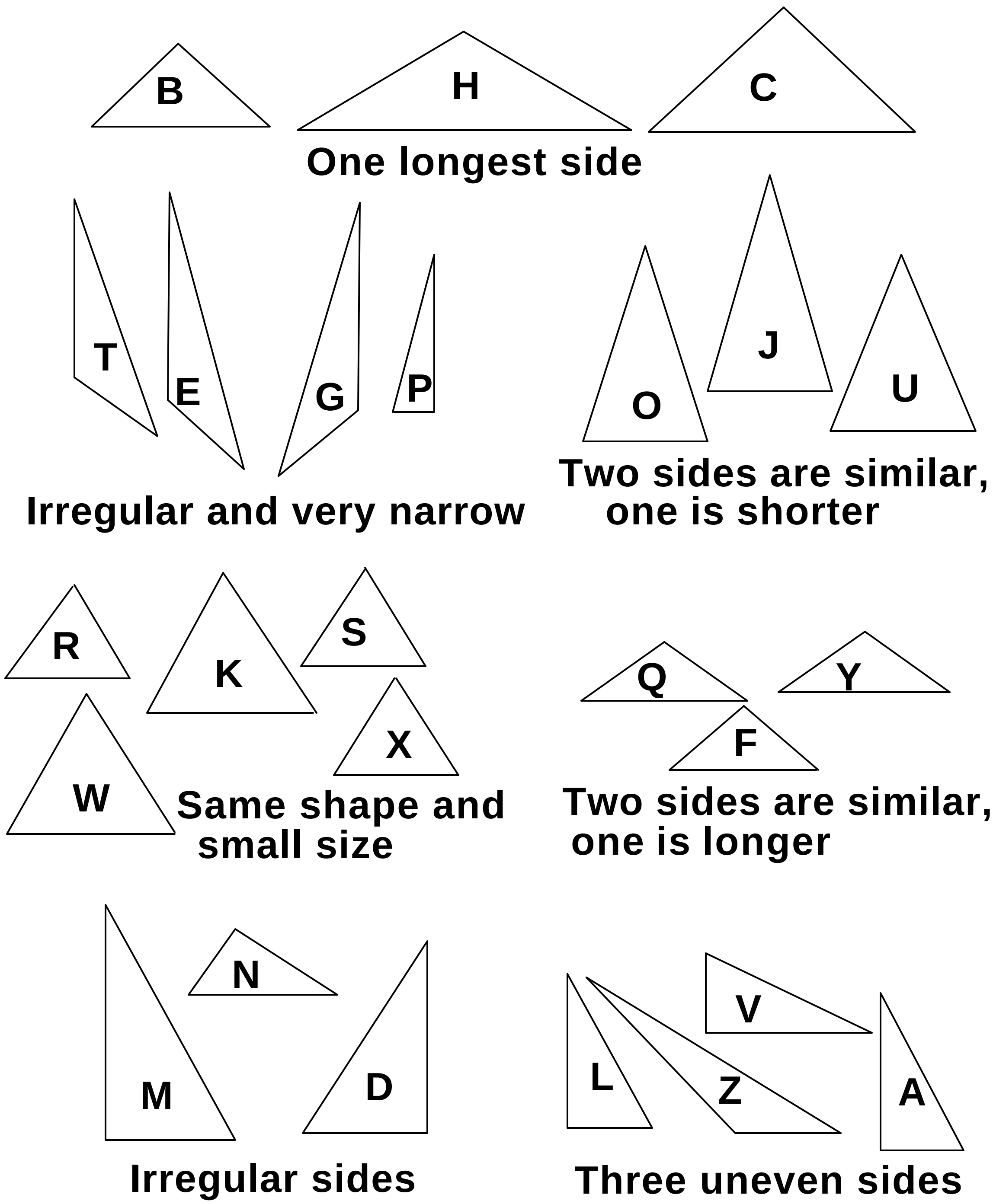


Scalene

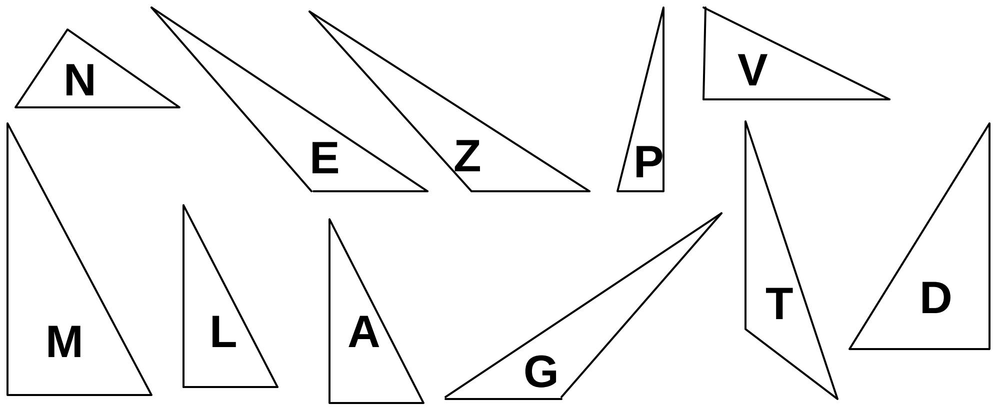


Isosceles

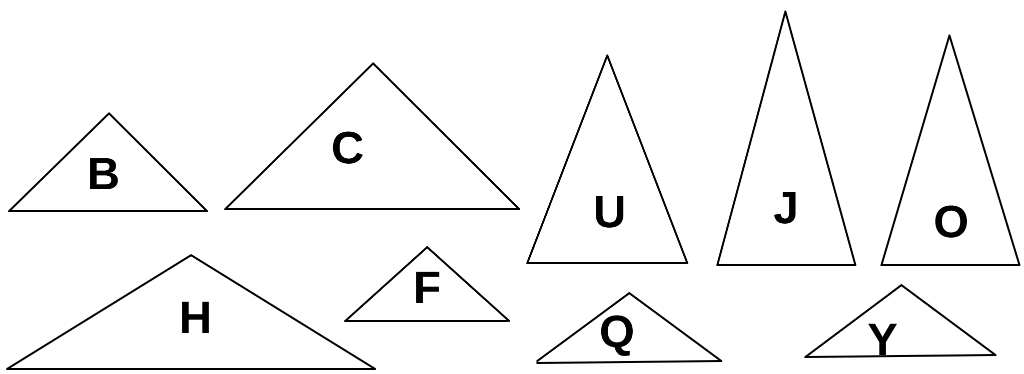
Sample Student Sort 5



Sample Student Sort 6

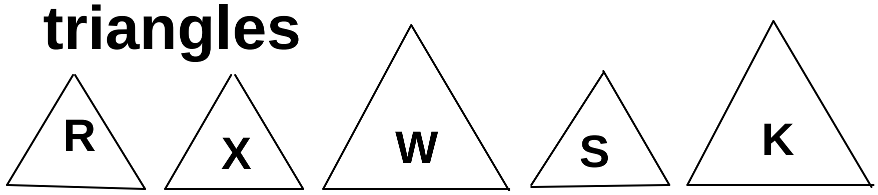


Every side has a different size



Two sides equal, third side smaller or larger

“Perfect” triangles



Topic: Quadrilaterals and their Properties

Description: Participants will explore quadrilaterals and their properties through the use of various manipulatives such as sorting pieces and geo-strips. The sequence of activities is designed to facilitate an increase in a learner's van Hiele level of thinking about quadrilaterals from Level 1 to Level 3. First, the participants learn how to determine the van Hiele levels of their own students by analyzing how they sort a set of quadrilateral pieces. Then they play the game "What's My Rule?" to develop the ability to classify quadrilaterals by various attributes and to focus on more than one attribute at a time. The participants also construct parallelograms, rectangles, rhombi, and squares using D-stix, geo-strips, toothpicks, or other manipulatives and make observations while the figures are flexed (Level 2). Finally, the participants identify relationships between parallelograms, rectangles, rhombi, squares, trapezoids, kites, and darts through a lab that culminates in the creation of a quadrilateral family tree (Level 3).
Although these activities are presented with quadrilaterals, most of them are easily adapted to triangles and other polygons.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15

Activity: Quadrilateral Sort

Format: Small Group/Large Group

Objectives: After performing their own sorts, participants will be able to distinguish the way students at various van Hiele levels of geometric thought may sort quadrilaterals.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. You will need at least one set of quadrilaterals for every three participants.

Time Required: Approximately 20 minutes

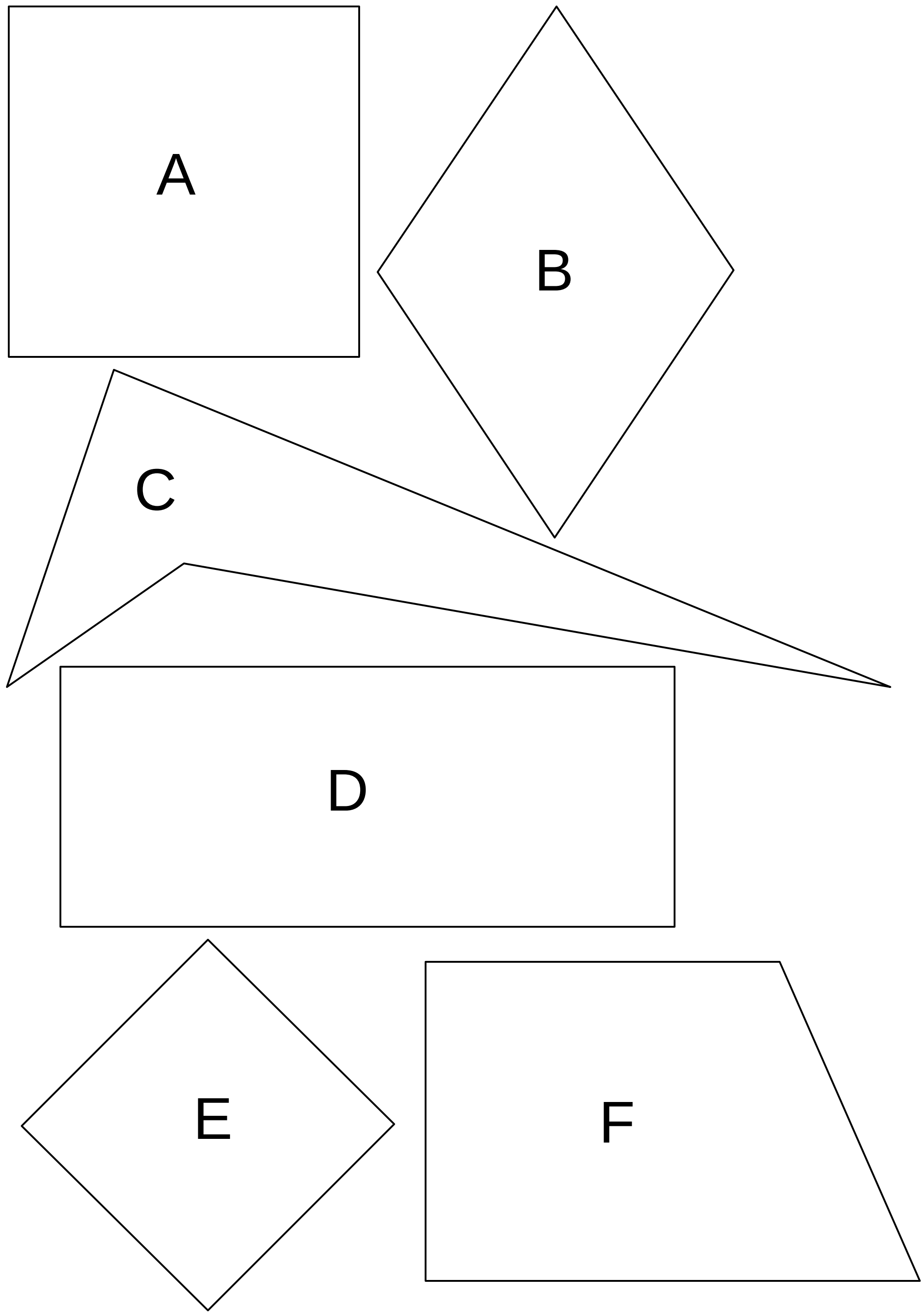
Directions:

1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, at least one set per three participants. Instruct the participants to lay out the pieces with the letters up. Do not call them quadrilaterals. Tell the participants that the objects can be grouped together in many different ways. For example, if we sorted the figures that make up the American flag (the red stripes, the white stripes, the blue field, the white stars), we might sort by color and put the white stripes and the stars together because they are white, the red stripes in another group because they are red, and the blue field by itself because it is the only blue object. Another way the flag parts could be grouped would be all the stripes and the blue field together because they are all rectangles and all the stars together because they are not rectangles. Have them sort the figures into groups that belong together, recording the letters of the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

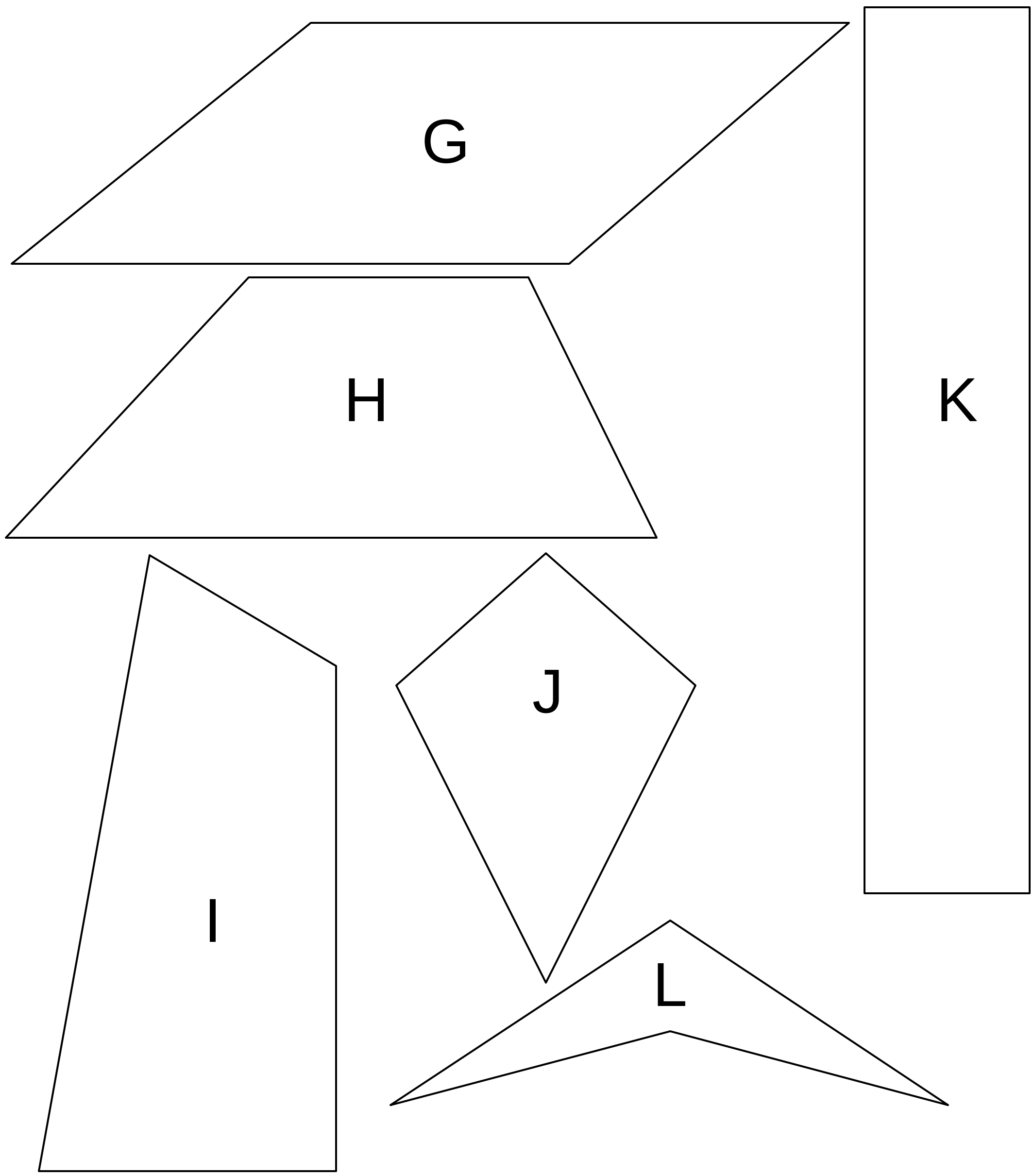
2) Ask the participants to describe their sorts. Have them compare their sorts with those of other groups.

3) Ask them how they think their students would sort these figures.

Quadrilateral Sorting Pieces



Quadrilateral Sorting Pieces
page 2



Activity: What's My Rule?

Format: Small Group/Large Group

Objectives: After playing the game, participants will classify quadrilaterals by various attributes. In children, this game develops the ability to attend to more than one characteristic of a figure at the same time.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). You will need at least one set of quadrilaterals for every three or four participants. "What's My Rule?" Activity Sheet

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into small groups. Distribute the sets of cut-out quadrilaterals, one set per group.

2) Display "What's My Rule?" Activity Sheet and review the rules of the game. One participant in each group is the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.

3) At any time, the players can call "stop" and guess the rule. After the correct rule identification, the player who figured out the rule becomes the sorter. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty. The winner is the first one to accumulate ten points.

WHAT'S MY RULE?

Rules

1. Choose one player to be the sorter. The sorter writes down a "secret rule" to classify the set of quadrilaterals into two or more piles and uses that rule to slowly sort the pieces as the other players observe.

2. At any time, the players can call "stop" and guess the rule. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty.

3. After the correct rule identification, the player who figured out the rule becomes the sorter.

4. The winner is the first one to accumulate ten points.

Activity: Quadrilateral Properties Laboratory

Format: Small Group/Large Group

Objectives: Participants will construct parallelograms, rectangles, rhombi, and squares, using D-stix, geo-strips, or toothpicks and marshmallows. Participants will identify the properties of the constructed figures.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15

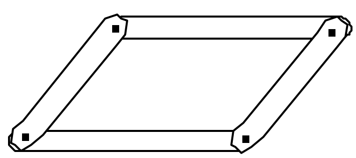
Materials: One of the following per participant: D-stix, geo-strips, or toothpicks cut into two different lengths and marshmallows; square corner (the corner of an index card or book); Types of Quadrilaterals Activity Sheet

Time Required: Approximately 20 minutes

Directions:

1) Divide the students into small groups and direct each group to experiment as you ask questions. Be sure to model constructing the quadrilaterals and flexing them.

2) Have the participants pick two pairs of congruent segments and connect them as shown below. Have them flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of the measures of the angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What do you notice about the opposite sides of this quadrilateral? (They remain parallel and congruent.)

A **parallelogram** is a quadrilateral with both pairs of sides parallel.

- What is the sum of the measures of the interior angles of this quadrilateral? (360°)

- What do you notice about the opposite angles? (Congruent)

Note to Trainer: Some participant will likely turn the strips so that they cross, forming two triangles. If no one does, you should. Ask if this figure is a polygon. Elicit from the group what the essential elements of a polygon are, i.e.,

a) composed of line segments
b) simple (the segments don't cross)
c) closed
d) lies in a plane (e.g., if you take a wire square and twist it so that it isn't flat, it is no longer a polygon)

3) Make one of the angles a right angle (You can use the square corner to check your accuracy.)
Ask:
   - What happens to the other angles? (They become right angles.)
   - Will this always be true when you make one angle of a parallelogram a right angle? (Yes)
   - How do you know? (The sum of the measure of the angles in a parallelogram is 360°. One angle measures 90°. Its opposite angle must measure the same or 90°. Subtracting these two angles from 360°, the remaining two angles, which are congruent since they are opposite angles in a parallelogram, must have a total measure of 180°. Therefore, each angle measure is 90°. Note: This is Level 3 thinking.)
   - Is it still a parallelogram? (Yes)
   - Is it still a quadrilateral? (Yes)
   - Is it still a polygon? (Yes)
   - What other name, besides polygon, quadrilateral, and parallelogram, can be given to it now? (rectangle)

A **rectangle** is a parallelogram with four right angles.

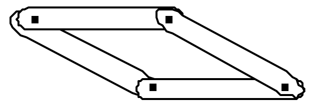
---

4) Make a parallelogram that has all four sides equal in length. What is another name for this parallelogram? (rhombus)

A **rhombus** is a parallelogram with four congruent sides.

---

5) Flex the figure to different positions.



Ask:

- What stays the same? (Lengths of the sides, the opposite sides are parallel, opposite angles are congruent, sum of angles, perimeter)
- What changes? (Size of angles, area, lengths of diagonals)
- What is the sum of the measures of the interior angles of this quadrilateral? (360°)
- What do you notice about the opposite angles? (Congruent)
- Is it still a quadrilateral? (Yes)
- Is it still a polygon? (Yes)

6) Make one of the angles of this rhombus a right angle, checking with your square corner.

   Ask:

   - What happens to the other angles? (All right angles)
   - Is it still a parallelogram? (Yes)
   - What other name, besides polygon, quadrilateral, parallelogram, and rhombus, can be given to this new figure? (square)

   A **square** is a parallelogram with four congruent sides and four right angles.

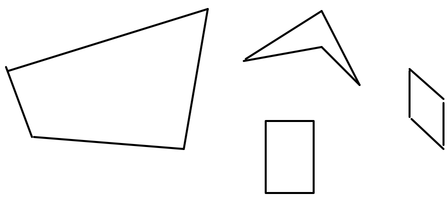
   Ask:

   - Is it a rectangle? (Yes)
   - How do you know? (It has four right angles.)

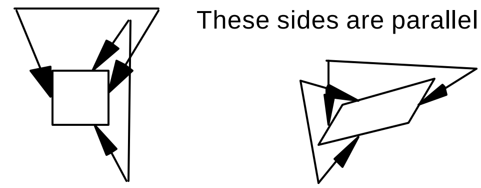
7) Distribute Types of Quadrilaterals Activity Sheet and discuss the definitions for quadrilateral, parallelogram, rectangle, rhombus, and square. Discuss the examples of each, noticing their orientations and how each example fits the definition even though they are not necessarily the stereotypical figure usually seen. Discuss the implications for teaching a Level 1 student who recognizes figures by comparing them to a known figure. This type of student might describe a rectangle by saying, “I know it’s a rectangle because it looks like a door.”

TYPES OF QUADRILATERALS

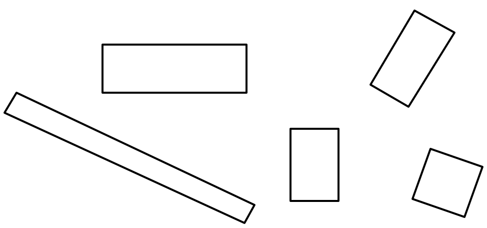
A **quadrilateral** is a **four**-sided polygon.



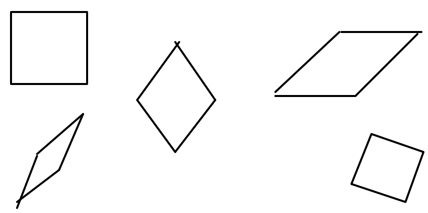
A **parallelogram** is a quadrilateral with both pairs of **opposite** sides **parallel**.



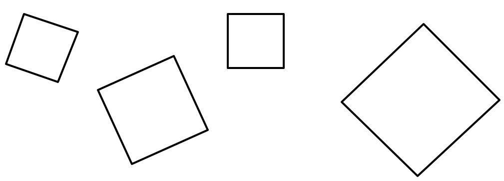
A **rectangle** is a quadrilateral with **four right angles**.



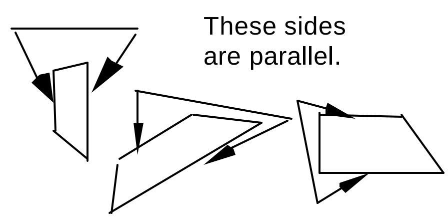
A **rhombus** is a quadrilateral with **four sides congruent**.



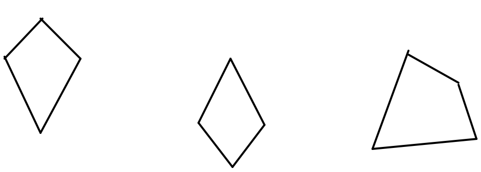
A **square** is a quadrilateral with **four right angles** and **four congruent sides.**



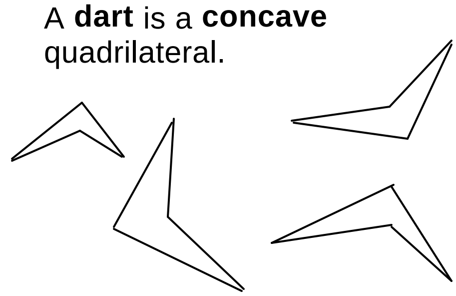
A **trapezoid** is a quadrilateral with **exactly one** pair of parallel sides.



A **kite** is a convex quadrilateral with two distinct pairs of **adjacent congruent sides**.



A **dart** is a **concave** quadrilateral.



Activity: Quadrilateral Sorting Laboratory

Format: Small Group/Large Group

Objectives: Participants will record which quadrilaterals meet the various descriptions listed in the properties table, determine which sets are identical and are subsets of one another, attach labels to each category, and create a quadrilateral family tree.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 4.15, 4.17, 5.14, 5.15

Materials: Quadrilateral Sorting Pieces Activity Sheet with quadrilaterals cut out and placed in a plastic baggy or manila envelope. (Use quadrilaterals from Quadrilateral Sort Activity). Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet, Quadrilateral Family Tree Activity Sheet

Time Required: Approximately 30 minutes

Directions:

1) Distribute Quadrilateral Sorting Laboratory Activity Sheet, Quadrilateral Table Activity Sheet and the Quadrilateral Family Tree Activity Sheet. Divide the participants into small groups and direct each group to experiment and answer the questions, using their quadrilateral sorting pieces.

2) After the participants have filled out the Quadrilateral Table Activity Sheet, have pairs of groups compare their answers, and reconcile any discrepancies.

3) Have the participants continue with Steps 5-10. Refer to the Quadrilateral Table as needed while discussing the results of #10.

4) For Step 11 the participants can construct the family tree as small groups or as a large group. Discuss various possibilities for the entries.

Quadrilateral Sorting Laboratory

Directions:

1) Spread out your quadrilateral pieces with the letters facing up so you can see them.

2) Find all of the quadrilaterals having four right angles. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.

3) Consider all of the quadrilaterals again. Find all of the quadrilaterals having exactly one pair of parallel sides. List them by letter alphabetically in the corresponding row of the Quadrilateral Table.

4) Continue in this manner until the Quadrilateral Table is complete.

5) Which category is the largest? What name can be used to describe this category?

6) Which lists are the same? What name can be used to describe quadrilaterals with these properties?

7) Are there any lists that are proper subsets of another list? If so, which ones?

8) Are there any lists that are not subsets of one another that have some but not all members in common? If so, which ones?

9) Which lists have no members in common?

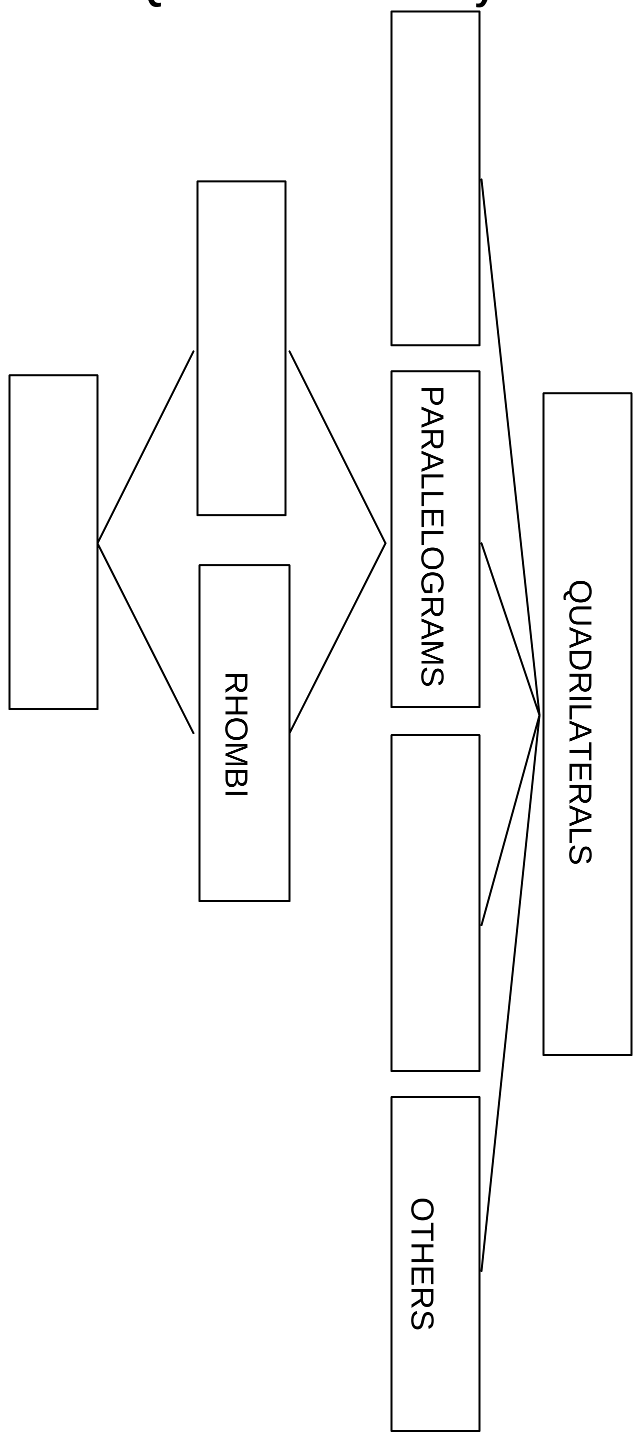
10) Label each of the categories in the Quadrilateral Table with the most specific name possible using the labels kite, quadrilateral, parallelogram, rectangle, rhombus, square, and trapezoid. For example, #1 - a quadrilateral that has four right angles is a rectangle. (Having four right angles is not enough to make it a square; it would need four congruent sides as well.)

11) Compare your results to that of the other groups. Then fill out the family tree by inserting the names kites, rectangles, squares, and trapezoids into the appropriate places on the diagram.

Quadrilateral Table

1. has four right angles
2. has exactly one pair of parallel sides
3. has two pairs of opposite sides congruent
4. has four congruent sides
5. has two pairs of opposite sides parallel
6. has no sides congruent
7. has two pairs of adjacent sides congruent, but not all sides congruent
8. has perpendicular diagonals
9. has opposite angles congruent
10. is concave
11. is convex
12. its diagonals bisect one another
13. has four sides
14. has four congruent angles
15. has four congruent sides and four congruent angles

Quadrilateral Family Tree



Elementary Geometry
Session 2

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Classification	Tibble	36	1.20		
	What's In the Box?	37	K.11, K.12, K.13, K.17, 1.20		Attribute blocks
	Missing Pieces	38	K.17, 1.20		Attribute blocks
	What's My Rule?	39	K.17, 1.20	What's My Rule?	Attribute blocks
	Twenty Questions Game	41	K.17, 1.20		Attribute blocks
	Who Am I? Game	42	K.17, 1.20	Clue Cards	Attribute blocks
	Differences – Trains and Games	50	K.17, 1.20	Differences Game Mat	Attribute blocks, game mat
	Hidden Number Patterns	53	K.17, 1.20	Hidden Number Sheets 1 and 2	Attribute blocks
	Attribute Networks	56	K.17, 1.20	Attribute Network Puzzle	Attribute blocks
Identifying Figures	Human Circle	59	K.11, K.12, 1.16, 1.17, 2.20, 2.22, 3.18, 4.17		String
	Geoboard Triangles and Quadrilaterals	60	K.11, K.12, 1.16, 1.17, 2.20, 2.22, 3.18	Geoboard Dot Paper	Geoboards, rubber bands, geoboard dot paper
	Figure Hunt	62	1.17		Electronic camera, computer, drawing software or Polaroid cameras and film; magic markers; poster board; and paste

Topic: Classification Using Attribute Materials

Description: Participants will explore the concept of classifying; a basic process of mathematical thinking that is essential to many concepts that are developed in the grades K-5 mathematics curriculum. Classification involves the understanding of relationships. Classification activities (observing likenesses and differences) can be presented through problem-solving situations and provide students with the opportunity to develop logical reasoning abilities. Logical reasoning skills and especially the meaningful use of the language of logic (if-then, and, or, not, all, some) are valuable across all areas of mathematics. An understanding of classification, or the recognition of the various attributes of items, is also an essential skill to patterning (extending, exploring, and creating patterns or sequences). These classification skills can be taught through a variety of materials; attribute blocks will be the manipulative used for this session.

The following description of attribute materials is taken from John Van de Walle's *Elementary and Middle School Mathematics: Teaching Developmentally*, 1997.

Attribute Materials:

Attribute materials are sets of objects that lend themselves to being sorted and classified in different ways. Natural or *unstructured* attribute materials include such things as seashells, leaves, the children themselves, or the set of the children's shoes. The *attributes* are the ways that the materials can be sorted. For example, hair color, height, and gender are attributes of children. Each attribute has a number of different *values*: for example, blond, brown, or red (for the attribute of hair color), tall or short (for height), male or female (for gender).

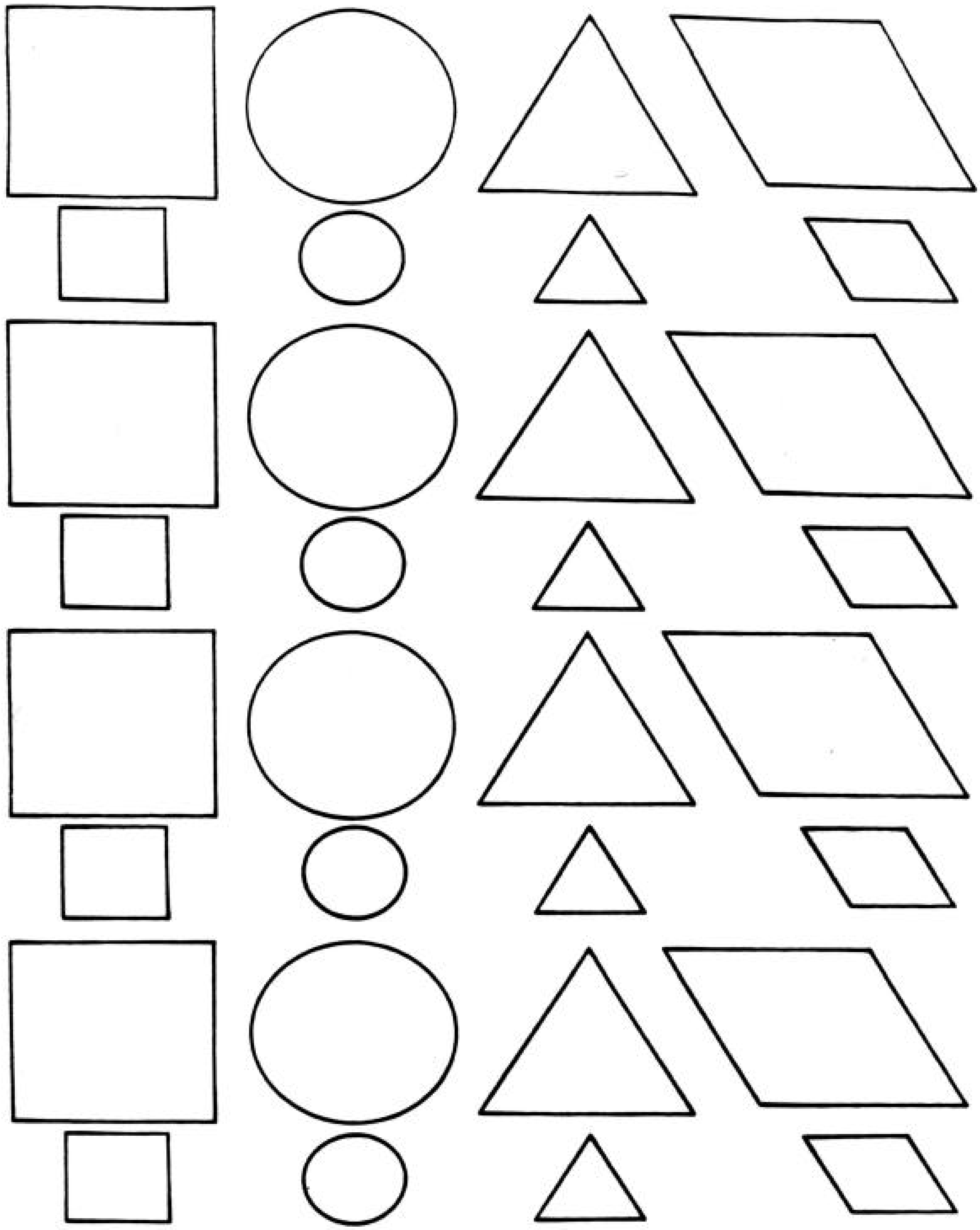
A *structured* set of attribute blocks has exactly one piece for every possible combination of values for each attribute. For example, several commercial sets of plastic attribute blocks have four attributes: color (red, yellow, blue), shape (circle, triangle, rectangle, square, hexagon), size (big, little), and thickness (thick, thin). In the set just described there is exactly one large, red, thin triangle, just as there is one each of all other combinations. The specific values, number of values, or number of attributes that a set may have is not important.

The value of using structured attribute blocks (instead of unstructured materials) is that the attributes and values are very clearly identified and easily articulated to students. There is no confusion or argument concerning what values a particular piece possesses. In this way we can focus our attention on the reasoning skills that the materials and activities are meant to serve. Even though a nice set of attribute blocks may contain geometric figures of different colors and sizes, they are not very good materials for teaching shape, color, or size. A set of attribute shapes does not provide enough variability in any of the shapes to help students develop anything but very limited geometric ideas. In fact, simple shapes, primary colors, and two sizes are usually chosen because they are most easily discriminated and identified by even the youngest of students (page 393).

Related SOL: K.11, K.12, K.13, K.17, K.18, 1.16, 1.20, 1.21, 2.25, 3.18, 3.24, 4.21, 5.20

Note:
On the following page you will find an Activity Sheet of the **32-Piece Attribute Blocks.** Copy the page on red, blue, green and yellow cover stock or construction paper and laminate the pages, if possible. Finally, cut out the shapes and place them in baggies before using them for instruction.

Attribute Blocks



Activity: Tibble

Format: Large Group

Objectives: Participants will use logical reasoning to identify specific attributes used to sort them into groups.

Related SOL: 1.20

Materials: None

Time Required: Approximately 5 minutes

Directions:

1) Select an attribute, such as the color of a participant's shirt, hair, or some other attribute. Do not tell the participants what has been selected. Call a participant's name and have him/her stand up and say, "You are a Tibble" if the participant has on the color of the shirt you're thinking of (or other attribute); otherwise, say "You are not a Tibble". Continue choosing participants that are Tibbles and not Tibbles. Have participants try to guess what makes a participant a Tibble or not a Tibble

2) Let participants take the lead and select a characteristic and determine who is and who is not a Tibble.

3) As the participants become more proficient in figuring out the selected attributes, involve two or more attributes in the determination of Tibbles.

Activity: What's In the Box?

Format: Large Group

Objectives: Participants will use logical reasoning to determine the number of pieces in the whole set of attribute blocks after asking questions and receiving information about a few items in the set.

Related SOL: K.11, K.12, K.13, K.17, 1.20

Materials: A standard 32 or 60 piece set of attribute blocks. Using a subset of attribute blocks such as all large or all thick can modify the level of difficulty of the activity. Before beginning this activity, check your set to be sure that it is complete.

Time Required: Approximately 15 minutes

Directions:

1) Tell participants, "This box (or bag) contains some materials." Shake it so the participants can hear. "I'd like you to ask me some questions with yes or no answers to figure out what is in the box." If the answer to the question is yes (i.e., Do you have something red in the box?), the trainer pulls out a block that has this attribute and will help the participants determine the number of items in the entire set (i.e., produce a red circle, then maybe a red triangle, to show there are other attributes besides color, etc.).

2) During the questions, ask:
   - How many pieces do you think I have left in the box?
   - How many pieces would the total set of attribute blocks contain? Why?
   - What are the characteristics (attributes) of this set of attribute blocks?
   - Is it possible to find a pair of attribute blocks that have neither size, color, thickness, nor shape in common?
   - Is each of the attribute blocks in your complete set unique?
   - How can you determine the number of pieces in the set of attribute blocks? (Answer: Multiply the number of each attribute.)

Activity: Missing Pieces

Format: Small Group

Objectives: Participants will visualize the whole set and divide it into a number of subsets based on the attributes in the set.

Related SOL: K.17, 1.20

Materials: Use standard 32 or 60 piece sets of attribute blocks. Using a subset of attribute blocks such as all large or all thick can modify the level of difficulty of the activity. Before beginning this activity, check your sets to be sure that they are complete.

Time Required: Approximately 15 minutes

Directions:

1) Divide the participants into small groups. Tell the participants to spread the attribute blocks on the table or desk. One participant removes a piece while the other participants look away. They are then asked to identify the missing piece without touching any of the pieces on the table. When the missing piece has been identified, the identifier removes the next piece, exchanging roles with the first participant. The activity is repeated.

2) After a few rounds of the activity, ask participants to describe the strategies used to organize the pieces.

Activity: What's My Rule?

Format: Small Group

Objectives: Participants will focus on more than one attribute at a time and use logical reasoning to determine how the set of attribute blocks were sorted.

Related SOL: K.17, 1.20

Materials: Use standard 32 or 60 piece sets of attribute blocks. Using a subset of attribute blocks such as all large or all thick can modify the level of difficulty of the activity. Before beginning this activity, check your sets to be sure that they are complete. What's My Rule? Activity Sheet

Time Required: Approximately 15 minutes

Directions: Divide the participants into small groups and have them spread the attribute blocks out. Distribute the What's My Rule? Activity Sheet and review the rules of the game. One participant, the sorter, thinks of a "secret rule" to classify the set of attribute blocks into two groups. The participant tells the rule to the trainer or writes it on a piece of paper without letting the other participants see it. The sorter uses that rule to slowly sort the pieces as the other participants observe. At any time, a player can call "stop" and guess the rule. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty. After the correct rule identification, the player who figured out the rule becomes the sorter. The winner is the first one to accumulate ten points.

WHAT'S MY RULE?

Rules

1. Choose one player to be the sorter. The sorter writes down a "secret rule" to classify the set of attribute blocks into two or more groups and uses that rule to slowly sort the pieces as the other players observe.

2. At any time, the players can call "stop" and guess the rule. The correct identification is worth five points. A correct answer, but not the written one, is worth one point. Each incorrect guess results in a two-point penalty.

3. After the correct rule identification, the player who figured out the rule becomes the sorter.

4. The winner is the first one to accumulate ten points.

Activity: Twenty Questions Game

Format: Small Group

Objectives: Participants play a game to develop skill at the strategy of elimination to reduce the set the quickest way in order to identify a specific element.

Related SOL: K.17, 1.20

Materials: Use a standard 32 or 60 piece set of attribute blocks. Using a subset of attribute blocks such as all large or all thick can modify the level of difficulty of the activity. Before beginning this activity, check your set to be sure that it is complete.

Time Required: Approximately 15 minutes

Directions:

1) This is a variation of the standard "Twenty Questions" game. Ask one participant to think of a block. The participant tells its name to the teacher or writes its name on a piece of paper without letting the other participants see it. The other participants, in turn, ask yes/no questions about the mystery block. After each question is answered, the participants move to one side those blocks that do not fit the clues already disclosed.

   A scorekeeper can count the number of questions asked. Participants try to find the mystery block using the fewest questions possible.

2) After a few games ask, "What is the best first question to eliminate the greatest number of blocks?" The participants may suggest, "Is the block four-sided?" This may not be the best first question, especially when the answer is no. Help the participants recognize that a strategy of eliminating the set by half is the quickest way to reduce the set and identify a specific element. In this game, if they are using the 32 piece set (Size: large or small; Color: red, yellow, green, or blue; Shape: square, rhombus, triangle, or circle), participants should learn to reduce the set by half each time as this strategy always provides the answer within five guesses (i.e., $32 = 2^5$). If they are using the 60-piece set (Size: large or small; Thickness: thick or thin; Color: red, yellow, or blue; Shape: square, rhombus, triangle, rectangle, or circle), the answer can be found within 7 guesses.

Activity: Who Am I? Game

Format: Small/Large Group

Objectives: Participants reinforce their understanding of attributes through a game where they use clues to identify a specific attribute piece.

Related SOL: K.17, 1.20

Materials: Use a standard 32 or 60 piece set of attribute blocks. Using a subset of attribute blocks such as all large or all thick can modify the level of difficulty of the activity. Before beginning this activity, check your set to be sure that it is complete. Clue Cards Activity Sheets (32-piece set – sheets 1-3) (60-piece set – sheets 4-6)

Time Required: Approximately 15 minutes

Directions:

1) This game can be played by individuals or teams. In the classroom with younger students, the teacher reads the clues. "Clue cards" should be prepared for participants. The players are shown a "clue card." The first player to discover the mystery piece is the winner.

2) Participants should be encouraged to develop their own problems to share with others. They may be used as an assessment.

Answers:

32 piece Attribute Set

1) Large, red rhombus
2) Small, red circle
3) Large, green rhombus
4) Small, green circle
5) Large, red triangle

60 piece Attribute Set

1) Large, thick, red rectangle
2) Small, thin, red circle
3) Large, thin, blue rectangle
4) Small, thin, yellow circle
5) Large, thin, red triangle
6) Large, thick , yellow rhombus

Clue Cards

(32 Piece Attribute Set)

(1)
I am large.
I am not yellow.
I have four sides.
I am not blue and not green.
I am not a square.
Who am I?

(2)
I am not large.
I am green or red.
I am not four sided.
I have no corners.
I am not green.
Who am I?

Clue Cards
(32 Piece Attribute Set)

(3)
I do not fit in a round hole.
I have four corners.
I am not red.
I am large.
I am green.
I am not square.
Who am I?

(4)
I am lost, help me find myself.
When you find me, hold me in your hand.
I am small.
I am not blue.
I am not square.
I am green.
I will roll off the table.
Who am I?

Clue Cards

(32 Piece Attribute Set)

(5)
I am blue or large or square.
I am not green.
I am small or a triangle.
I am red or blue.
I am not a circle.
I am blue or large.
I am not blue.
Who am I?

(6)
Write Your Own!

Who am I?

Clue Cards

(60 Piece Attribute Set)

(1)
I am large and not a square.
I am not yellow.
I have four sides.
I am not blue or thin.
I am not a rhombus.
Who am I?

(2)
I am not large.
I am yellow or red.
I am not four sided.
I have no corners.
I am not yellow or thick.
Who am I?

Clue Cards

(60 Piece Attribute Set)

(3)
I do not fit in a round hole.
I have four corners.
I am not red.
I am large.
I am green.
I am blue and thin.
I am not square.
I am not a rhombus.
Who am I?

(4)
I am lost, help me find myself.
When you find me, hold me in your hand.
I am small. I am not blue.
I am not square or thick.
I am yellow.
I will roll off the table.
Who am I?

Clue Cards

(60 Piece Attribute Set)

(5)
I am blue or thin or square.
I am not yellow.
I am small or a triangle.
I am red or blue.
I am not a circle.
I am blue or large.
I am not blue.
Who am I?

(6)
I am not small or not blue.
I am thick or a triangle.
I am a square or a rhombus.
I am yellow or small.
I am not red or large.
I am not a square.
I am large or not a rhombus.
Who am I?

Activity: Differences - Trains and Games

Format: Small Group

Objectives: Participants will identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) as they create difference trains and play games where they must identify the number of differences.

Related SOL: K.17, 1.20

Materials: Attribute blocks, Differences Game Mat

Time Required: Approximately 30 minutes

Directions:

1) Ask participants to compare blocks in terms of their differences and similarities. Hold up a block and ask the participants to hold up a block that differs in one way. Repeat this with several blocks, and then ask them to hold up a block that differs in two ways, then in three ways. At the same time, ask the participants to hold up blocks that are similar in two, one, or no ways.

2) Tell the participants that "Difference Trains" have engines and cars. Place the large, red circle on the table as the engine of the train. Cars are to be sequentially attached to the train according to the given rule. Start with the rule that the car to be attached must differ from the preceding car by a single attribute - by one difference. That is, if the engine is a large, red circle, then there are a variety of possibilities that could be attached as the cars; for example, a small, red circle or a large, yellow circle. Have participants identify all of the possibilities. Ask the participants "Why could the small, blue square not be the first car attached to the large, red circle engine?" Taking turns with their partners, have the participants build a train at least 20 cars long, verbalizing the difference as the next car is put into place.

3) Ask the participants "Could you have built a train using all of the attribute blocks? Try it?"

4) Two-Difference Variation: Have the participants start with the same engine. This time attach a car that differs from the car to which it is attached by two-differences. Ask the participants "Could you have built a train using the 30 large pieces before using any small pieces? Try it!"

5) Three-Difference Variation: Have the participants agree on the attribute block to be the engine. Build a train so that the adjacent cars will differ by exactly three differences.

6) Differences is a game for two players or two teams on a four-by-four game mat. The blocks are randomly divided equally between the two players or teams. A turn consists of placing a block on the game mat. THE ONLY RULE is that a block must differ from its horizontal and vertical neighbors in exactly one way. The first player who cannot place a block loses. Extension: a block must differ from its neighbors horizontally, vertically, and diagonally. In Game Two, the block must differ in two ways.

Difference Game Mat

Games: Blocks must differ from their horizontal and vertical neighbors in:

- exactly one way (Game 1)
- exactly two ways (Game 2)

Activity: Hidden Number Patterns

Format: Small Group

Objectives: Participants will identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) as they examine trains of attribute blocks on Hidden Number Sheets..

Related SOL: K.17, 1.20

Materials: Hidden Number Sheets 1 and 2.

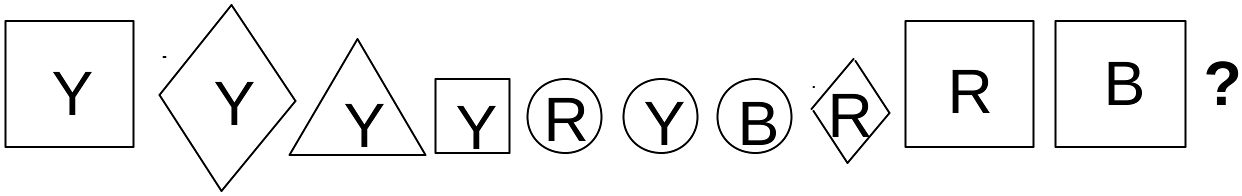
Time Required: Approximately 10 minutes

Directions:

1) Have the participants identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) in the Hidden Number Sheets 1 and 2.

2) Have participants create difference trains where they develop a number pattern for the number of differences in their trains.

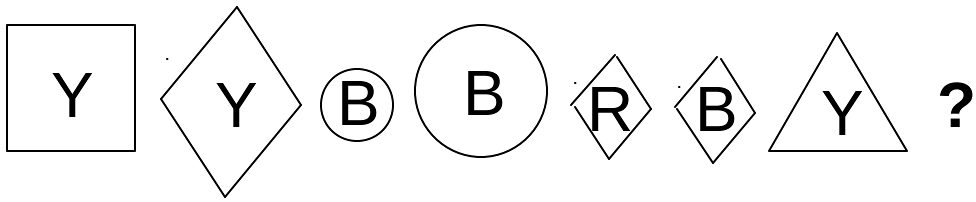
Hidden Number Sheet #1



What is the number pattern (of differences) associated with this train?

How many different blocks could be placed after the large blue square?

Hidden Number Sheet #2



What large block would you place after the large yellow triangle?

Why did you select that block?

Are there any other blocks that could immediately follow the large yellow triangle?

What number pattern (of differences) did you discover in the train?

Activity: Attribute Networks

Format: Small Group/Individual

Objectives: Participants complete an attribute network problem to demonstrate their understanding of differences. This may be used as an assessment of the participants' understanding of one-, two-, and three-differences.

Related SOL: K.17, 1.20

Materials: Attribute Network Puzzle Activity Sheet

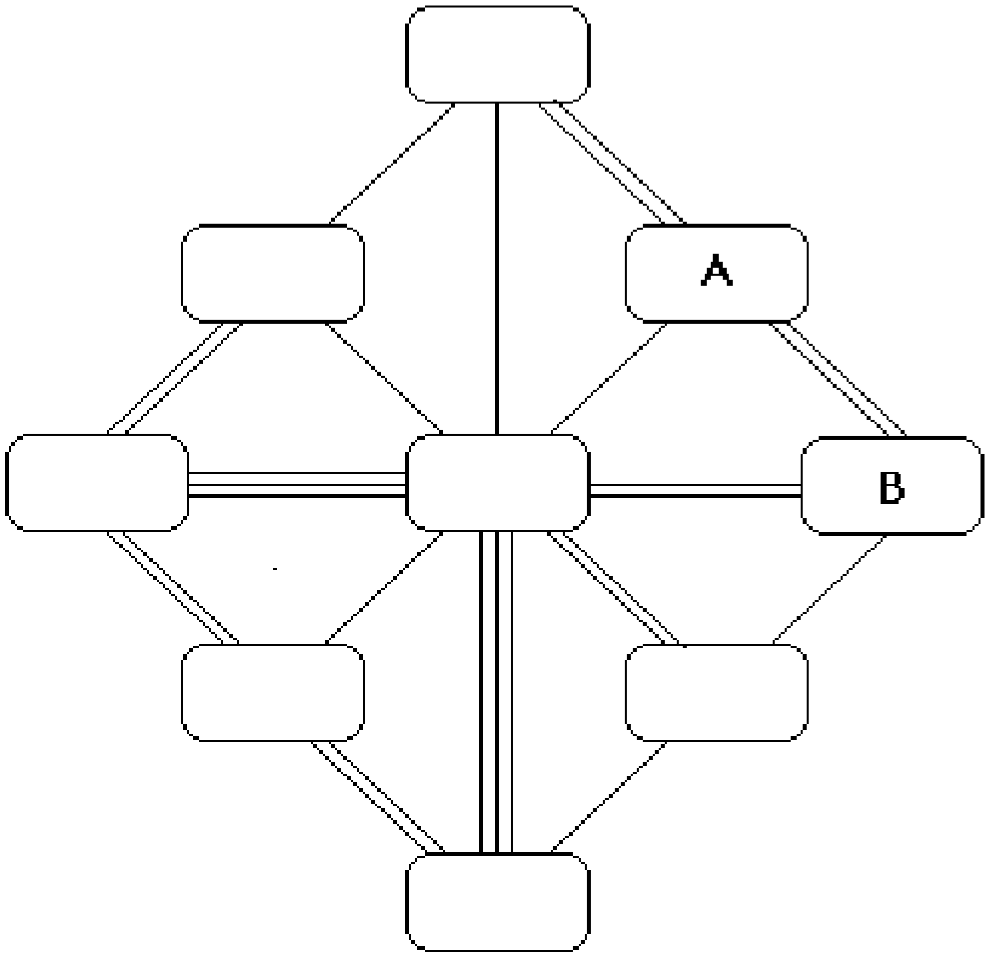
Time Required: Approximately 20 minutes

Directions:

1) Distribute Activity Sheets. Place an attribute block on one of the regions. Place another attribute block in an adjacent region that differs from the first piece by as many variables (i.e., color, shape, size, thickness) as there are lines connecting the regions. For example the piece in Region B must differ from the piece in Region A by exactly two attributes.

2) Once you have created a solution that works, write your answers in each block.

Attribute Network Puzzle



Topic: Identifying Figures

Description: Participants will explore figures by creating a human circle, creating figures on geoboards, and conducting a figure hunt.

Related SOL: K.11, K.12, 1.16, 1.17, 2.20, 2.22, 3.18, 4.17, 4.17

Activity : Human Circle

Format: Large Group

Objective: Participants will develop a definition of a circle and a sphere after creating a human circle.

Related SOL: K.11, K.12, 1.16, 1.17, 2.20, 2.22, 3.18, 4.17

Materials: Enough pieces of string, each the same length (5-10 feet, depending on the space available), for each participant but one; chalk (optional)

Time required: Approximately 15 minutes

Directions:

1) Clear a space larger than twice the length of the cut string or go outside to the playground.

2) Choose one participant to be the center.

3) Have this center person hold the ends of all the strings in one hand, making a fist with all the strings coming out of the top. This person should crouch down, with his/her arm held over his/her head.

4) Have every other person take an end of the string and back up so the string is taut, spacing themselves around the center person in all directions.

5) (Optional) Draw a circle with chalk on the ground or floor to approximate the circle created by the humans.

6) Discuss the circle as the set of all points in a plane that are the same distance from the center.

7) Extend the idea to the set of all points in space that are the same distance from a center point. Contrast a circle and a sphere.

Activity : Geoboard Triangles and Quadrilaterals

Format: Large Group

Objective: Participants will identify various polygons by their number of sides and angles, regardless of their orientation or size.

Related SOL: K.11, K.12, 1.16, 1.17, 2.20, 2.22, 3.18

Materials: Geoboards, rubber bands, geoboard dot paper

Time required: Approximately 15 minutes

Directions:

1) Distribute geoboards, several rubber bands, and a sheet of geoboard dot paper to record their results to participants.

2) Ask the participants to make a three-sided figure on their geoboard and to record it on their dot paper.

3) Display the geoboards on the chalkboard ledge. Have the participants sort them in various ways (e.g., those having a square corner, those having two equal sides).

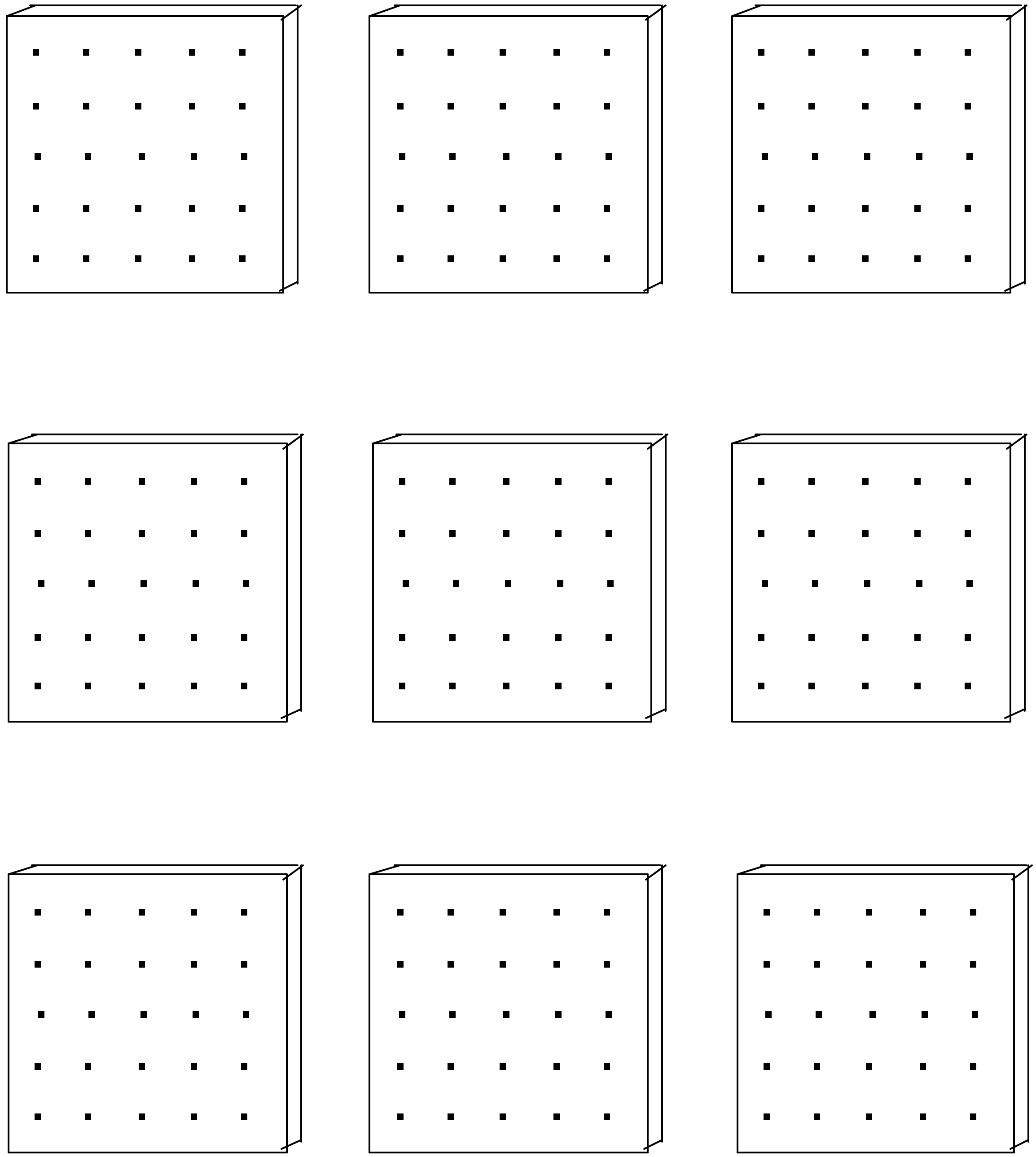
4) If none of the triangles made are isosceles right triangles, challenge the participants to make a three-sided figure with a square corner and two sides equal in length. Can more than one be made? How are they alike? How are they different?

5) Ask the participants to make a figure that has four sides and four square corners and to record it on their dot paper.

6) Have the participants make as many **different** figures as they can that have four sides and four square corners. Have them copy these figures onto their dot paper. Discuss how these figures are alike and how they are different.

7) Have the participants change the figure on their geoboard from a four-sided figure with square corners to a four-sided figure without any square corners. Copy it onto dot paper and compare to the four-sided figures with square corners. Have the participants describe what they have made.

Geoboard Dot Paper



Activity: Figure Hunt

Format: Small Group/Large Group

Objective: Participants will identify various objects in their environment that depict triangles, rectangles, squares, and circles.

Related SOL: 1.17

Materials: Electronic camera, computer, and drawing software or Polaroid cameras and film, magic markers, poster board, paste

Time required: Approximately 30 minutes

Directions:

1) Instruct the participants in the use of the camera.

2) Organize the participants into groups of two or three. Assign each group a figure, such as, triangle, rectangle, square, or circle.

3) Each group should identify and photograph various objects they find within a specified area that depict their assigned shape.

4) If an electronic camera was used, download the images and print them.

5) Have the participants outline their figures with the magic marker and create a poster or mural by gluing the figures to the poster board and labeling them.

6) Display and discuss the results.

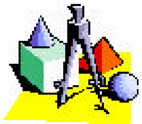
Elementary Geometry
Session 3

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Spatial Relationships	Square It	65	K.11, K.12, 1.16, 2.13	Square It	Playing board, grid paper, colored 1"-square markers
	Pick Up the Sticks	67	K.11, K.12, 1.16	Pick Up the Toothpicks	Toothpicks
	Partition the Square	69	K.11, 1.16, 1.21, 2.25, 3.18	Partition the Square	Paper
	Cutting Square Puzzles	71	3.18, 4.14, 4.16	Cutting Square Puzzles	Multicolored construction paper, scissors, glue
Tangrams	Make Your Own Tangrams	74	K.11, 1.16, 3.18	Directions for Making Tangrams	Paper, scissors
	Area and Perimeter Problems/Tangrams	76	2.12, 2.13, 4.13, 5.8, 5.10	Area and Perimeter with Tangrams	Tangram set
	Spatial Problem Solving with Tangrams	79	3.18, 5.10	Problem Solving with Tangrams Sheets 1, 2	Tangram set, puzzles
Symmetry	Butterfly Symmetry	83	2.21, 3.20, 4.21, 5.20		Construction paper, pencils, scissors, magazines, paint, crayons or markers
	Copy Cat	84	2.21, 3.20, 4.21, 5.20		Paper, ruler, Mira
	Recover the Symmetry	86	2.21, 3.20, 4.21, 5.20		Paper, ruler, Mira
	Folded Figures	89	2.21, 3.20, 4.21, 5.20	Folded Figures	Paper, ruler, Mira
	Symmetry and Right Angles in Quadrilaterals	91	1.16, 2.21, 3.20, 4.15, 5.14, 5.15	Quadrilateral Patterns Sheets 1, 2, 3	Scissors, file cards
	Origami: Making a Square	96	K.11, 1.16, 2.21, 3.18, 3.20	Folding a Square	Scissors, non-square pieces of paper
	Origami: Making a Heart	98	K.11, 1.16, 2.21, 3.18, 3.20	Making a Heart	Scissors, non-square pieces of paper

Topic: Spatial Relationships

Description: To build spatial visualization skills, students need a wide variety of experiences, including building and dissecting figures from different perspectives. In these activities, participants will explore spatial relationships by playing a strategy game with squares, solving a match stick triangle puzzle, partitioning squares into smaller squares, and using square dissection puzzles.

Related SOL: K.11, K.12, K.13, 1.16, 2.13, 2.25, 3.18, 4.13, 4.14, 4.16, 5.8, 5.10



Activity: Square It

Format: Small Group

Objectives: Participants will recognize squares and gain practice in visualization. As an extension, students will determine the area of a square by counting the number of square units needed to cover it.

Related SOL: K.11, K.12, 1.16, 2.13

Materials: Playing board, Square It Activity Sheet or 8 x 11 one-inch grid paper, and colored one-inch square markers of two different colors.

Time Required: Approximately 15 minutes

Directions:

1) Players choose who starts. Play rotates clockwise.

2) Player places a marker of his or her color on a vacant box on the playing board. Players alternate placing markers.

3) The winner is the player to first recognize a SQUARE on the board where all four corners are his or her color. Players check for "squareness" by counting the lengths of the sides. Winning squares may range from 2 x 2 to 7 x 7.

4) (Optional) The winning player must state the area of the winning square.

Square It

Activity: Pick Up the Toothpicks

Format: Small Group

Objectives: Participants will recognize triangles and gain practice in visualization by picking up varying numbers of toothpicks to yield a certain number of triangles.

Related SOL: K.11, K.12, 1.16

Materials: 11 toothpicks per participant, Pick Up The Toothpicks Activity Sheet

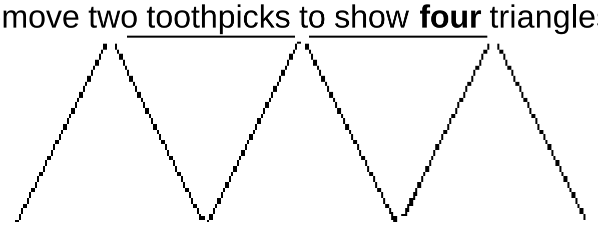
Time Required: Approximately 10 minutes

Directions:

1) Pass out 11 toothpicks per participant. Tell them to use the physical materials suggested as they work through the problems posed. They should not consider this activity to be concerned with mental, spatial-visualization skills -- they should actually use the materials.

2) Discuss the directions on the Activity Sheet. Eleven toothpicks are arranged as shown to give five triangles. For each problem begin with the original 11-stick configuration and

   A. remove two toothpicks to show **three** triangles;
   B. remove one toothpick to show **four** triangles;
   C. remove three toothpicks to show **three** triangles;
   D. remove two toothpicks to show **four** triangles.



3) Be sure to discuss the fact that all the sides don't have to be the same length in order for a three-sided figure to be a triangle.

Pick Up the Toothpicks

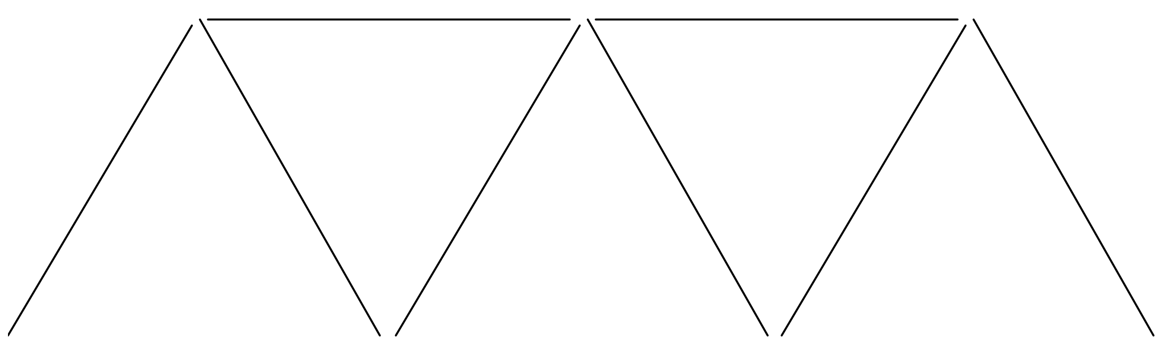
Eleven toothpicks are arranged as shown to give five triangles. For each problem, begin with the original 11-stick configuration and...

A. remove two toothpicks to show **three** triangles;

B. remove one toothpick to show four triangles;

C. remove three toothpicks to show **three** triangles;

D. remove two toothpicks to show **four** triangles.



Activity: Partition the Square

Format: Individual /Large Group

Objectives: Participants will partition squares into 7-15 smaller squares and will explain how they know that the smaller figures are really squares.

Related SOL: K.11, 1.16, 1.21, 2.25, 3.18

Materials: Partition the Square Activity Sheet, scratch paper

Time Required: Approximately 30 minutes

Directions:

1) Distribute the Partition the Square Activity Sheet. Explain to the participants that they are to divide each square into smaller squares and that there are many ways to determine each number of squares. Use the two sample partitions to point out that the squares don't have to be the same size, just that four sides of each square must be congruent, and that overlaps will not count.

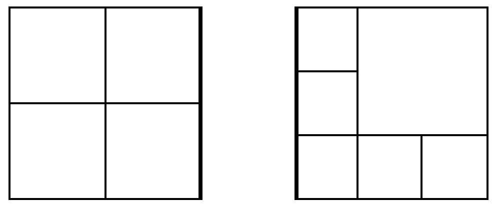
2) Circulate around the room, referring participants to the two samples if they need assistance. Also, look for non-square rectangles and remind the participants that all four sides of a square are congruent.

3) After they have had a few minutes to work, ask the participants to share their solutions. Start out with labels 7 - 15 and ask for a volunteer to do each one. Ask participants to add their method if they have a different way of partitioning than the one shown.

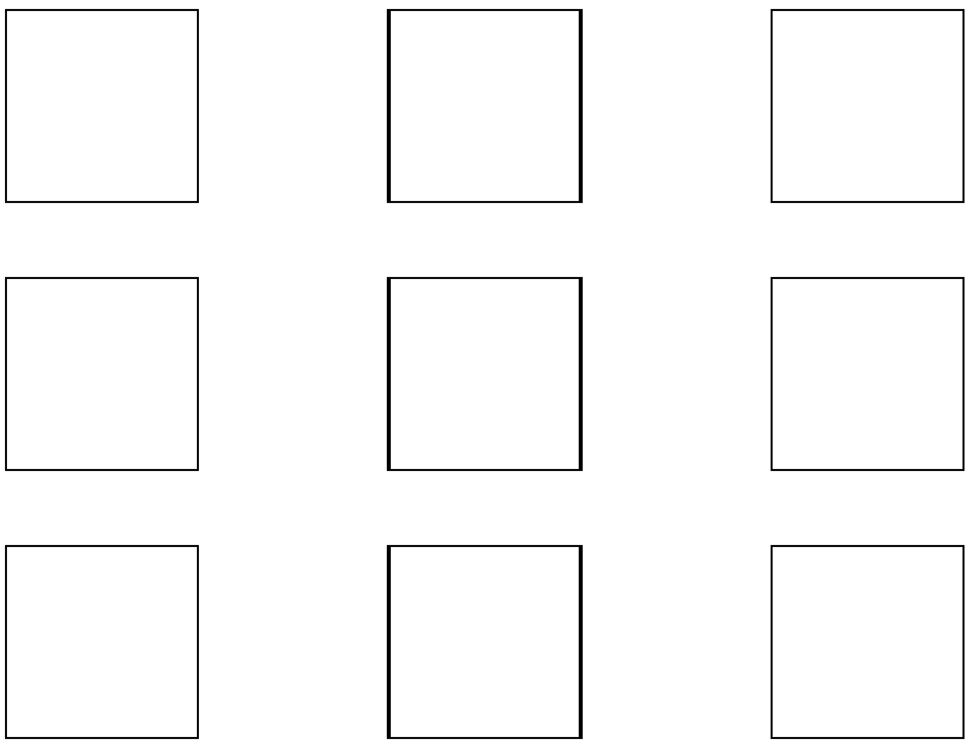
4) After all solutions have been shared by the participants, challenge the group to justify that each really is composed of squares. Tell them that you will allow them to assume that angles that look like right angles are right angles. Perhaps the simplest way of justifying four congruent sides in each of the drawn squares is to think of the original square as a unit square and then label the sides accordingly.

Partition the Square

A square can be partitioned into squares in more than one way. Shown below are squares partitioned into 4 smaller squares and 6 smaller squares.



Use these partitioning ideas to find ways to partition the nine squares below into 7 - 15 smaller squares.



Activity: Cutting Square Puzzles

Format: Small Group

Objective: Participants will investigate the relationships between the parts of a square to the whole square, of acute angles to right angles to obtuse angles, and of the diagonals to the sides of a square.

Note: This activity was adapted from "Geometry - A Square Deal for Elementary School" by Marcia E. Dana in *Learning and Teaching Geometry, K-12*, the 1987 NCTM Yearbook.

Related SOL: 3.18, 4.14, 4.16

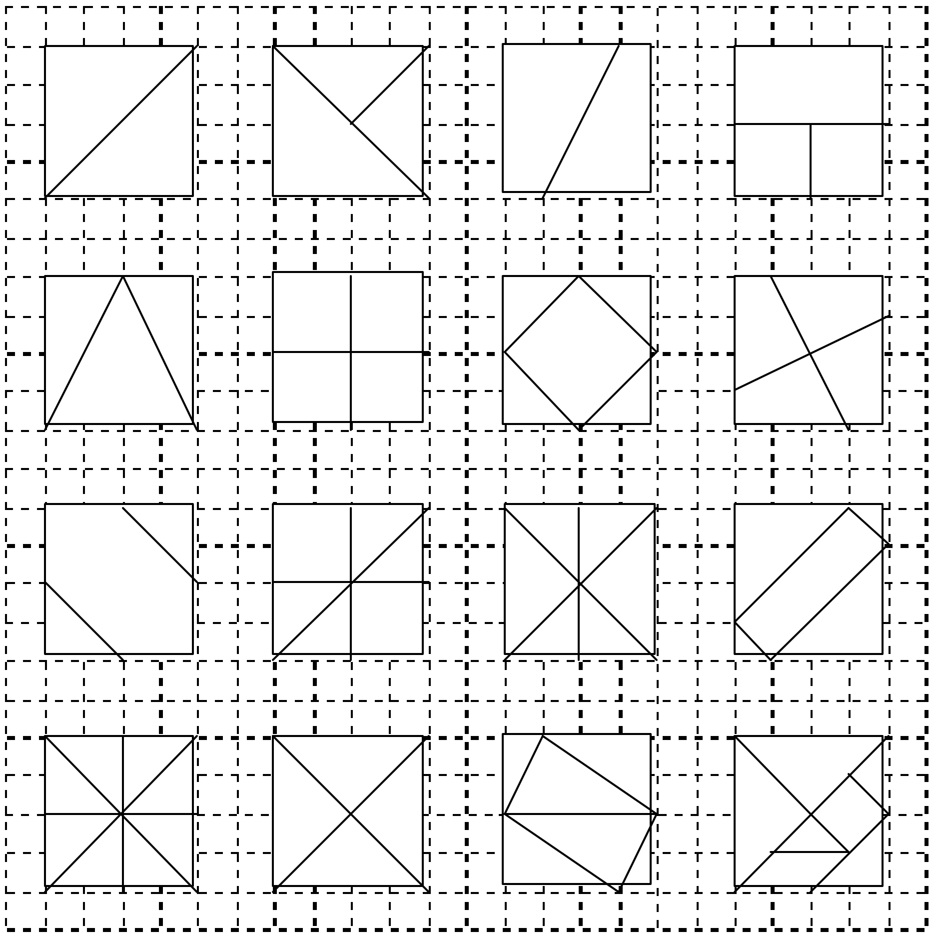
Materials: Construction paper or tag board puzzles of various colors in envelopes, scissors, a sheet of paper with the proper size square outlines on it, glue, Cutting Square Puzzles Activity Sheet

Time required: Approximately 30 minutes

Directions:

1) Make squares of various colors of construction paper or tag board and cut each square a different way as shown on the activity sheet. Put each puzzle in an envelope so that the participants may try as many as they wish. Note: The puzzles are shown on grid paper for clarity. They should be made on construction paper or tag board for participant use.

Possible Square Puzzles



Note that the last square is a tangram.

Topic: Tangrams

Description: Participants will make their own tangrams and will use them to explore area and perimeter relationships in geometric figures. They will engage in problem solving puzzles using tangrams.

Related SOL: K.11, 1.16, 2.12, 2.13, 3.18, 4.13, 5.8, 5.10

Activity: Make Your Own Tangrams

Format: Small Group

Objectives: Participants will construct their own tangrams and identify properties of the seven tangram pieces.

Related SOL: K.11, 1.16, 3.18

Materials: Paper suitable for folding such as copy paper (one sheet per student), scissors, one set of overhead tangrams, Directions for Making Tangrams Activity Sheet with directions for making tangrams

Time Required: 30 minutes

Directions:

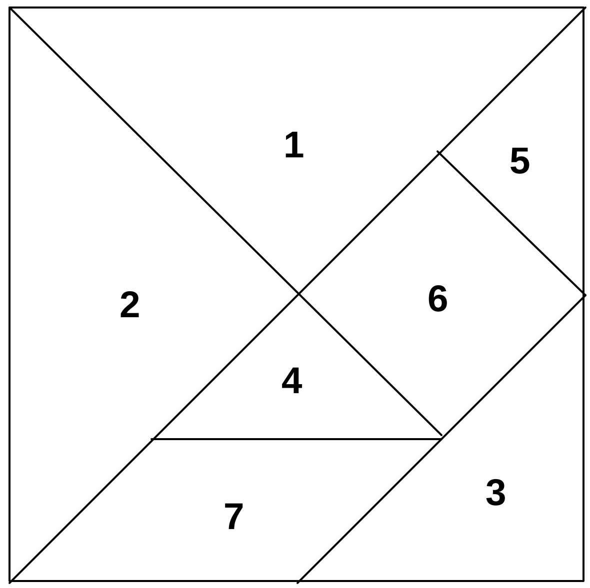
1) Distribute directions for making a set of the seven tangram pieces (Directions for Making Tangrams Activity Sheet) or give the directions orally for participants to follow individually as you make a master set as a demonstration.

2) Ask participants to make a square out of all seven tangram pieces.

3) Have participants label the pieces by number as indicated in the diagram below. They should also identify each by the name of the figure. Put a set of overhead tangrams on the overhead projector and discuss:
   - Identify each tangram piece by the name of the figure.
   - Which figures are congruent? How do you know?
   - Which triangles are similar? How do you know? Can you write a proportion to express the relationship?

4) Have participants find the measure of each angle of each figure. They should trace each figure and label the angle measures.

Directions for Making Tangrams



1. Fold the lower right corner to the upper left corner along the diagonal. Crease sharply. Cut along the diagonal.

2. Fold the upper triangle formed in half, bisecting the right angle, to form Piece 1 and Piece 2. Crease and cut along this fold. Label these two triangles "1" and "2".

3. Connect the midpoint of the bottom side of the original square to the midpoint of the right side of the original square. Crease sharply along this line and cut. Label the triangle "3".

4. Fold the remaining trapezoid in half, matching the short sides. Cut along this fold.

5. Take the lower trapezoid you just made and connect the midpoint of the longest side to the vertex of the right angle opposite it. Fold and cut along this line. Label the small triangle "4" and the remaining parallelogram "7".

6. Take the upper trapezoid you made in Step 4. Connect the midpoint of the longest side to the vertex of the obtuse angle opposite it. Fold and cut along this line. Label the small triangle "5" and the square "6".

Activity: Area and Perimeter Problems with Tangrams

Format: Individual/Small group

Objectives: Participants will express the area of each tangram piece as a fraction of the large composite square and will find the area of each tangram piece by using the square piece as the unit and then using the small triangle as the unit. This will require the application of the Pythagorean Theorem.

Related SOL: 2.12, 2.13, 4.13, 5.8,5.10

Materials: A set of tangrams for each participant, Area and Perimeter with Tangrams Activity Sheet

Time Required: 30 - 40 minutes

Directions:

1) Give participants the Area and Perimeter With Tangrams Activity Sheet. Make sure they also have a set of tangrams. Ask them to work alone or in small groups to complete the tasks outlined on the Activity Sheet. Circulate around the room helping participants who need assistance.

2) Invite participants to the overhead projector to describe how they found the answers.

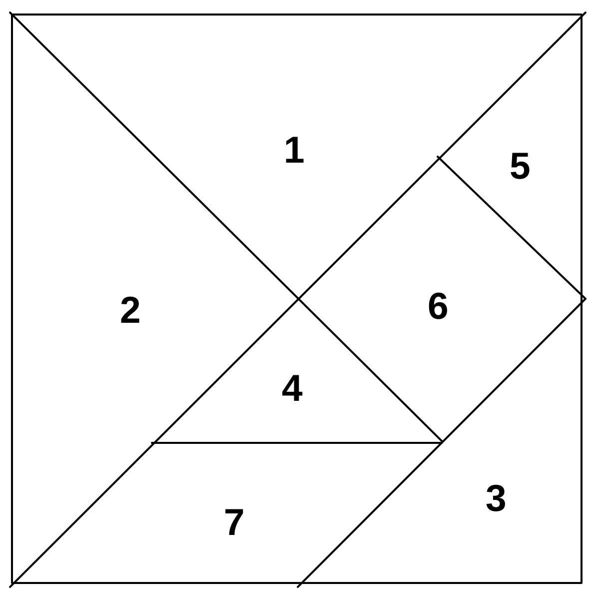
Area and Perimeter With Tangrams

1. If the area of the composite square (all seven pieces -- see below) is 1 unit, find the area of each of the separate pieces.

piece #	area
1	
2	
3	
4	
5	
6	
7	

2. If the smallest triangle (piece #4 or #5) is the unit for area, find the area of each of the separate pieces in terms of that triangle.

Piece #	area
1	
2	
3	
4	
5	
6	
7	



3. If the smallest square (piece #6) is the unit for area, find the area of each of the separate pieces in terms of that square. Enter your findings in the table below.

4. If the side of the small square (piece #6) is the unit of length, find the perimeter of each piece and enter your findings in the table.

piece #	area	perimeter
1		
2		
3		
4		
5		
6		
7		

Activity: Spatial Problem Solving with Tangrams

Format: Independent/Small Group

Objectives: Participants will create geometric figures with a subset of the tangram set. They will solve puzzles by arranging the set of seven tangrams to form the pictures given to them.

Related SOL: 3.18, 5.10

Materials: A set of tangrams for each student; Spatial Problem Solving with Tangrams Activity Sheet, Tangram Puzzles Activity Sheet

Time Required: Variable, allow 30 minutes to get started. Participants may work independently over a period of a week or so and turn in solutions at a later session.

Directions: Distribute Activity Sheets and have participants work individually or in small groups to solve the tangram puzzles.

Spatial Problem Solving with Tangrams

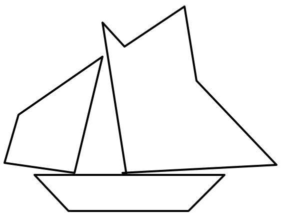
Use the number of pieces in the first column to form each of the geometric figures that appear in the top of the table. Make a sketch of your solution(s). Some have more than one solution while some have no solution.

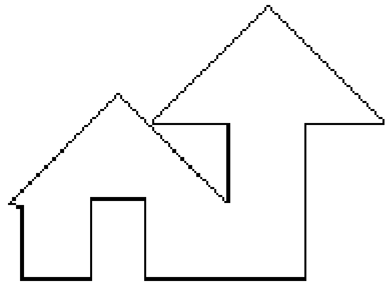
Make These Polygons

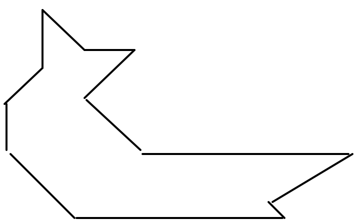
Use this many pieces	Square	Rectangle	Triangle	Trapezoid	Trapezoid	Parallel -ogram
2						
3						
4						
5						
6						
7						

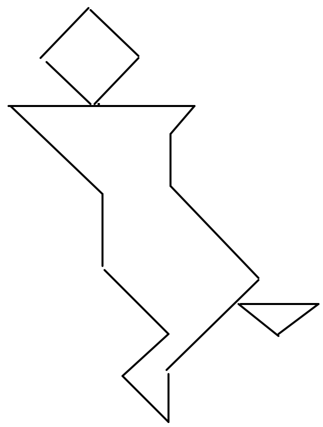
Tangram Puzzles

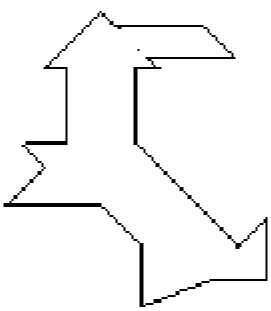
Can you make these figures with the seven tangram pieces? Make a sketch of your solutions.

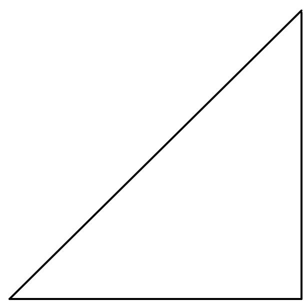
1. 

2. 

3. 

4. 

5. 

6. 

Design your own tangram picture. Trace the outline and give it a name. Submit the outline and a solution key.

Topic: Symmetry Using Paper Folding, Pattern Blocks, Miras, and Origami

Description: Participants will explore the concept of symmetry, a phenomenon that occurs frequently in nature. Butterflies, leaves, blossoms, and even the human body are symmetric. Since symmetric forms are pleasing to the human eye, many artists and architects incorporate symmetry in their designs. Even many national flags and corporate logos exhibit symmetry.

There are two types of symmetry: **line symmetry** (also called reflectional or mirror symmetry) and **point symmetry** (also called rotational symmetry).

Pattern Blocks: *Pattern blocks* are a set of blocks that consist of figures: a hexagon (usually yellow), a trapezoid (usually red) that is one-half the size of the hexagon, a rhombus (usually blue) one-third the size of the hexagon, a triangle (usually green) one-sixth the size of the hexagon, and another rhombus (usually tan). As their name implies, they are used in creating patterns, but they are also excellent for teaching tessellations, fractions, and several other concepts. In this section, we will use them for exploring symmetry.

Mirrors or Miras: Small mirrors are useful in checking line symmetry. If you are worried about using glass mirrors with children, there are several commercially available alternatives such as polished metal mirrors or Miras (a rectangle of translucent red plastic with a beveled edge at the bottom and attached rectangular sides to stabilize the reflector and serve as handles).

Origami: Origami is the Japanese art of paper-folding. Many of the figures that can be made are symmetric. Special origami paper may be purchased from teacher supply stores or most mail order math supply companies. A sheet of origami paper will be square, white on one side, and colored on the other side. It will make a sharp crease when folded properly and has stiffness to it so that it will hold the figure into which it is folded. If you are not able to locate origami paper, heavy wrapping paper may be cut into squares and used. In a pinch, patty paper works, but it is not a very good substitute because it does not hold its figure and does not have an obvious wrong side and right side.

Related SOL: K.13, 1.16, 2.21, 3.18, 3.20, 4.21, 5.19, 5.20

Activity: Butterfly Symmetry

Format: Small Group

Objectives: Participants will create symmetric cutouts and will identify the lines of symmetry.

Related SOL: 2.21, 3.20, 4.21, 5.20

Materials: Construction paper, pencils, scissors, magazines, paint, crayons or markers

Time Required: Approximately 30 minutes

Directions:

1) Divide the participants into small groups. Have them fold a piece of paper once and cut out any figure, beginning and ending on the folded edge. Have them unfold the paper and make observations about their figures. They should notice that there are matching parts. Tell them that since all the parts match when folded on a straight line, we say that the figure has line symmetry and that the fold is the line of symmetry. Ask the participants if their figures can be folded any other way so that the parts match.

2) Now repeat the folding and cutting activity using two folds. Again have the participants describe the figures they have created. Ask how many ways the figure can be folded so that the two halves match. (When the paper is folded twice, the cut figure will have two lines of symmetry with four matching parts.)

3) Have the participants find pictures in magazines and objects in the classroom that have line symmetry. Questions you might ask: What do we mean when we say that something has symmetry? What does a line of symmetry do? What does it show us? Could we draw a line of symmetry anywhere on a symmetrical figure?

4) (An additional activity for students in the classroom). Use paper, scissors, and markers, crayons, or paint to create symmetrical butterflies for display on the bulletin board. As a class, find the lines of symmetry on each butterfly and discuss.

Activity: Copy Cat

Format: Small Group

Objectives: The participants will create a symmetrical figure, piece by piece, using pattern blocks to verify the figure's symmetry.

Related SOL: 2.21, 3.20, 4.21, 5.20

Materials: About 30 pattern blocks per group, paper, ruler, Mira, optional overhead pattern blocks

Time Required: Approximately 30 minutes

Directions:

1) Give each participant a piece of paper and some pattern blocks. Have them make a fold line with the paper and then spread it out flat or draw a thick vertical line down the center of the paper. Have them place a pattern block on the paper so that one of its edges lies along the fold line. Another participant should place another pattern block on the other side of the line so that it forms a design that is symmetrical along the line. Explain that the block should be placed so that when you trace the blocks and fold the paper along the line, the two traced figures will cover each other exactly. Trace around the two blocks, fold the paper, and verify that the design is symmetrical.

   Example:

2) Unfold the paper, and put the two blocks back in the same positions. Have another person in the group place a block on one side of the line of symmetry so that it is touching at least one of the blocks already placed there. Have another person place a block on the other side to "balance" that block and keep the design symmetrical. Continue to add blocks to the design and check its symmetry.

3) Change the rules so that participants place two or more pattern blocks on one side of the design during each turn.

4) Challenge the participants to make a design that is symmetrical without using a fold line or drawn line. Have them use a Mira to check the symmetry of their design.

5) Using the Mira, have the participants find the lines of symmetry for their pattern blocks. For this activity, the Mira is placed on top of the block and moved around until the image matches the block. Have the participants trace the block on paper and record the lines of symmetry of each block. Discuss the results with the whole group.

Activity: Recover the Symmetry

Format: Small Group

Objectives: Participants will explore symmetry concepts using pattern blocks by verifying that a design has line symmetry.

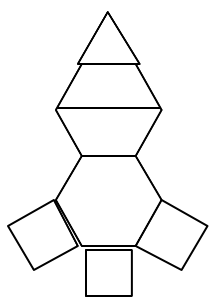
Related SOL: 2.21, 3.20, 4.21, 5.20

Materials: About 30 pattern blocks per group, paper, ruler, Mira, optional overhead pattern blocks, Rules for Recover the Symmetry Activity Sheet

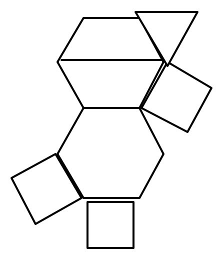
Time Required: Approximately 15 minutes

Directions:

1) Before meeting with the group, build two identical symmetrical figures on the overhead projector. Then move two blocks in one of the figures to make it asymmetrical. For example:



Symmetrical Design



Asymmetrical Design

2) Display both figures and identify them as symmetrical and asymmetrical, respectively. Identify the line of symmetry on the symmetrical design.

3) Have the participants suggest how they might move two blocks in the asymmetrical design to make it symmetrical. Test the results.

4) Distribute the Recover the Symmetry Rules. Discuss the rules with participants.

5) After playing several rounds of Recover the Symmetry, discuss the games with the class. Other questions for discussion are:

- What was easier, creating the original design or recovering the symmetry? Why?
- Did you ever find more than one line of symmetry in a design? If so, describe how this happened.
- What helped you decide how to move the blocks to recover the symmetry?
- When you made an asymmetrical figure symmetrical, did you recreate the original figure or did you find a new one? Why do you think this happened?

Recover the Symmetry Rules

1. This is a game for two players.

2. Together, the players build a symmetrical design using 12 pattern blocks. It should be symmetrical in both shape and color.

3. Players can use a Mira to check the symmetry of the design. They should locate all lines of symmetry in the design.

4. One player looks away while the other player moves 3 blocks so that the design is not symmetrical.

5. The player who wasn't looking then tries to move 3 blocks to make the design symmetrical again.

6. After the symmetry has been recovered, players talk over these questions with each other: Is the new design symmetrical? Were the same blocks moved to recover the symmetry? Is the final design the same as the original one?

7. Play several rounds of Recover the Symmetry and discuss your games with the group. Other questions for discussion are:

   - What was easier, creating the original design or recovering the symmetry? Why?
   - Did you ever find more than one line of symmetry in a design? If so, describe how this happened.
   - What helped you decide how to move the blocks to recover the symmetry?
   - When you made an asymmetrical figure symmetrical, did you recreate the original figure or did you find a new one? Why do you think this happened?

Activity: Folded Figures

Format: Small Group

Objectives: Participants will construct the other half of pattern block designs by imagining that the design is unfolded or that a mirror is placed along a given line of symmetry.

Related SOL: 2.21, 3.20, 4.21, 5.20

Materials: About 30 pattern blocks per group, paper, ruler, Mira, Folded Figures Activity Sheet, optional overhead pattern blocks

Time Required: Approximately 15 minutes

Directions:

1) Divide the participants into small groups. Distribute Folded Figures Activity Sheet to each individual and about 30 pattern blocks to each group. Tell the participants that the designs they see on the Activity Sheet are pattern block designs that have been folded along the line of symmetry. It is their task to imagine what each design would look like unfolded and then sketch it. First of all, recreate the design using the pattern blocks as in the previous activity. Then sketch what they think the other side looks like. Check it by removing the pattern blocks and placing a Mira along the line of symmetry. If Miras or mirrors are unavailable, the figure may be folded along the line of symmetry and held up to a light to check for congruence.

2) Have the participants complete the remaining figures and check their symmetry. Display the results on the overhead.

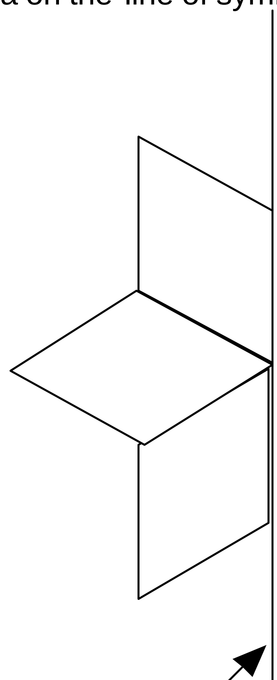
3) Ask the participants to check to see if any of the designs have more than one line of symmetry.

4) Have the participants create an original design having only one line of symmetry. Have them check the symmetry using a Mira or by folding along the line of symmetry.

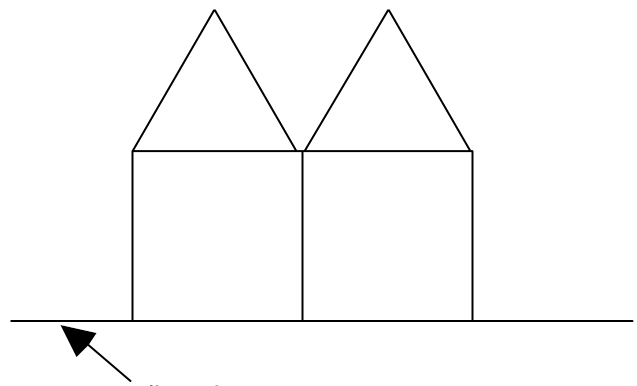
5) Have the participants create an original design having two lines of symmetry and check it by folding along each line of symmetry in turn.

Folded Figures

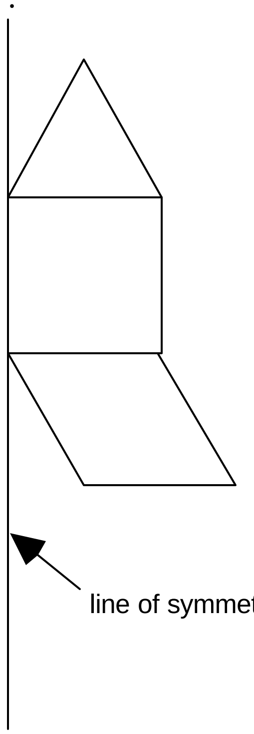
These pattern block designs were folded along the indicated line of symmetry. Imagine what each design would look like unfolded and sketch it. Check your work by placing a Mira on the line of symmetry.



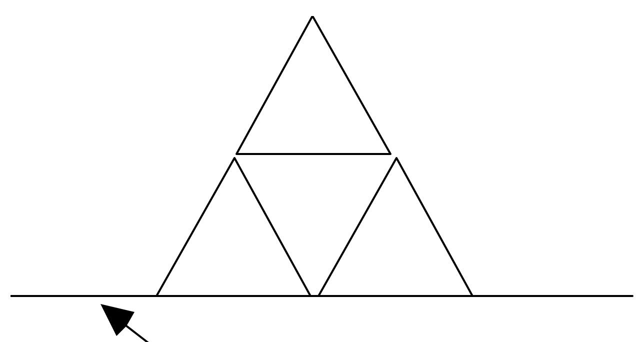
line of symmetry



line of symmetry



line of symmetry



line of symmetry

Activity: Symmetry and Right Angles in Quadrilaterals

Format: Small Group

Objectives: Participants will fold various cut-out quadrilaterals to determine how many lines of symmetry each has and use square corners to determine how many right angles each has.

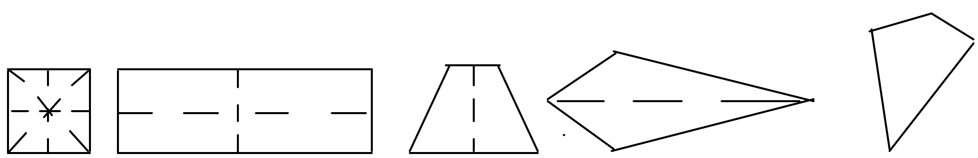
Related SOL: 1.16, 2.21, 3.20, 4.15, 5.14, 5.15

Materials: Scissors, Quadrilateral Patterns Activity Sheets 1, 2, and 3 or cut-out quadrilaterals, square corners (e.g., a piece of paper or a file card)

Time Required: Approximately 15 minutes

Directions:

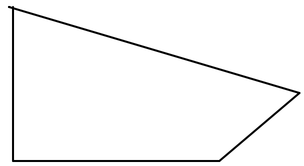
1) Give each participant Symmetry and Right Angle Activity Sheets 1, 2, and 3 and a pair of scissors and have them cut out each quadrilateral or give each participant a set of cutout quadrilaterals. Have them find as many ways as possible to fold the figures in half so the halves match, i.e., find all the lines of symmetry. Discuss the results.



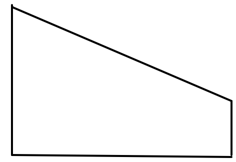
Four Two One None

2) Give each participant a square corner and the figures from Step 1. Ask them to find a quadrilateral that has an angle that is
   a) a right angle (the same as a square corner)
   b) greater than a right angle
   c) less than a right angle.

3) Add figures such as the following:



One right angle



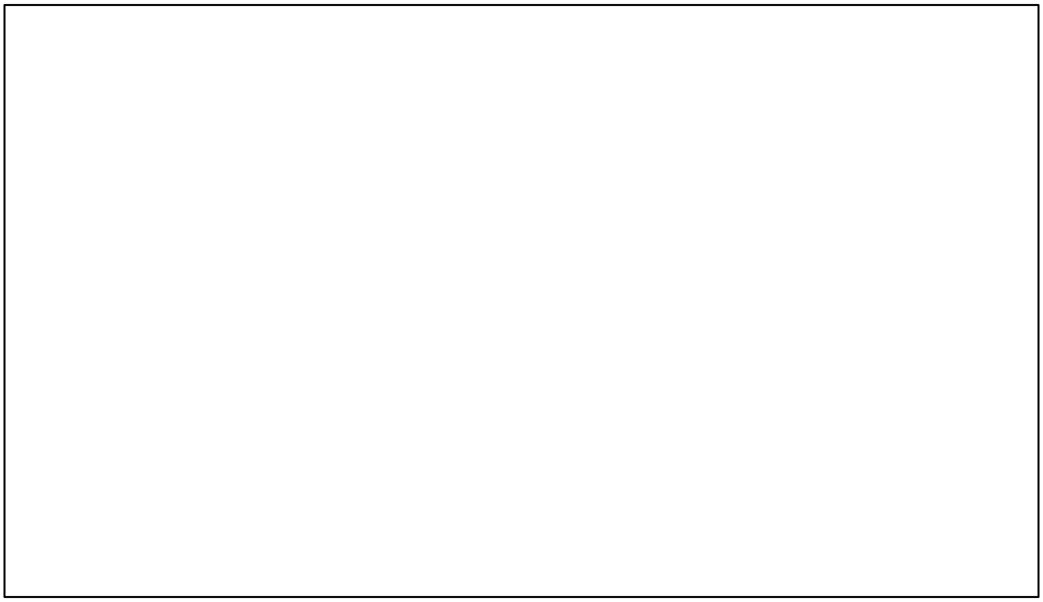
Two right angles

Ask the participants to find a quadrilateral that has exactly

a) one right angle,
b) two right angles, and
c) four right angles.

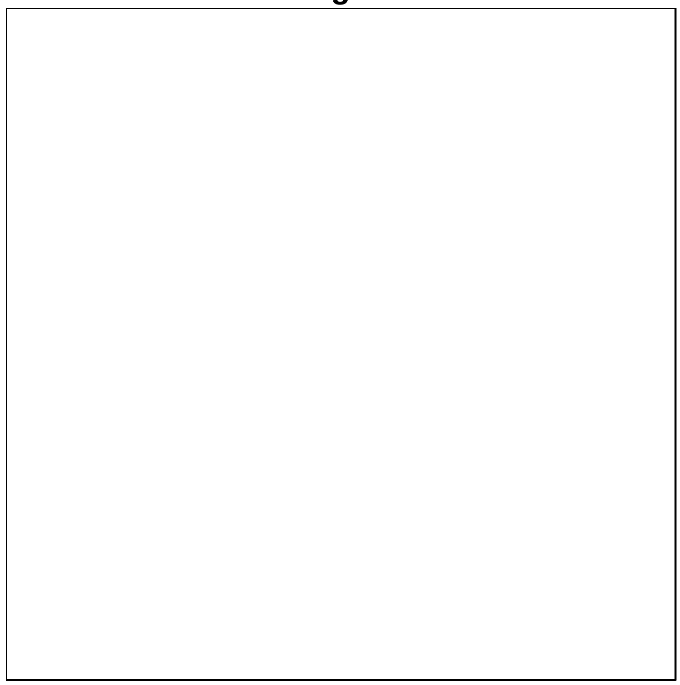
How many different quadrilaterals can they find that have four right angles? What are the special names of these quadrilaterals?

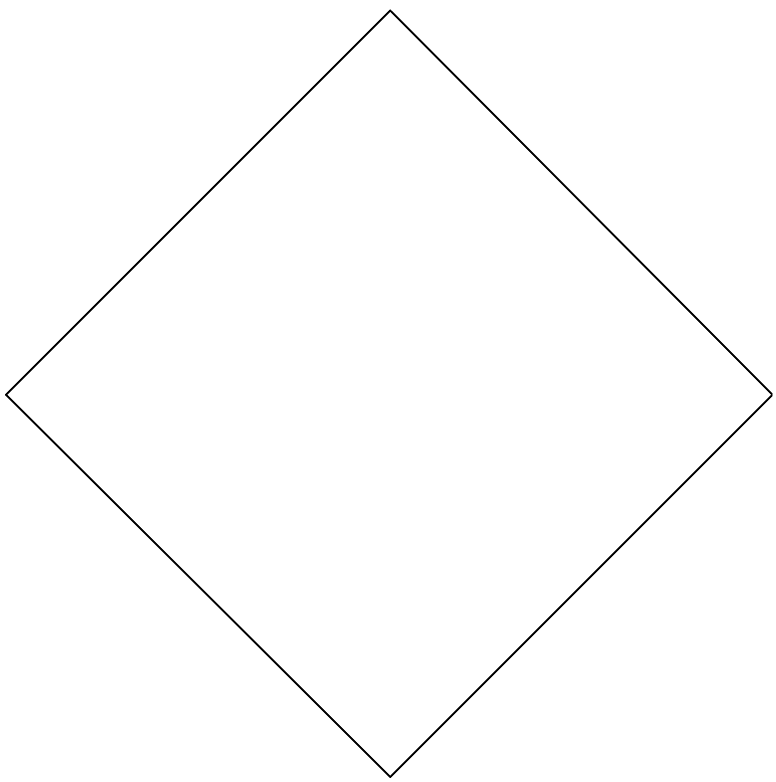
Quadrilateral Patterns



Quadrilateral Patterns

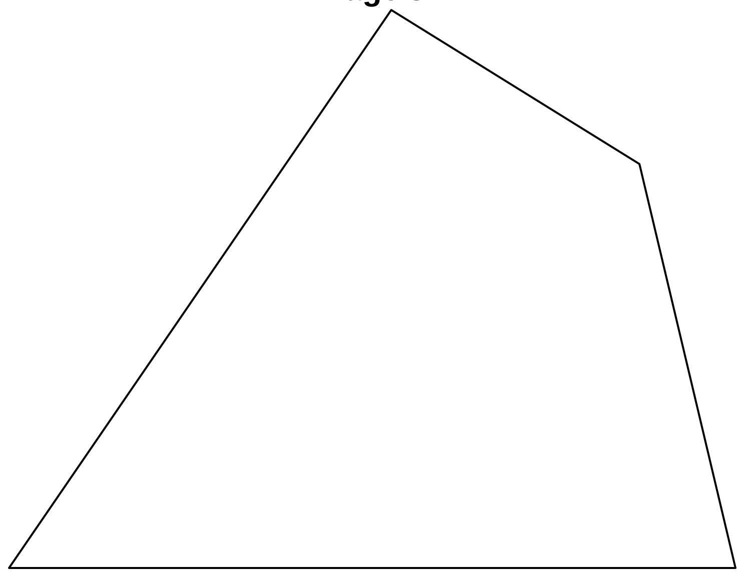
Page 2

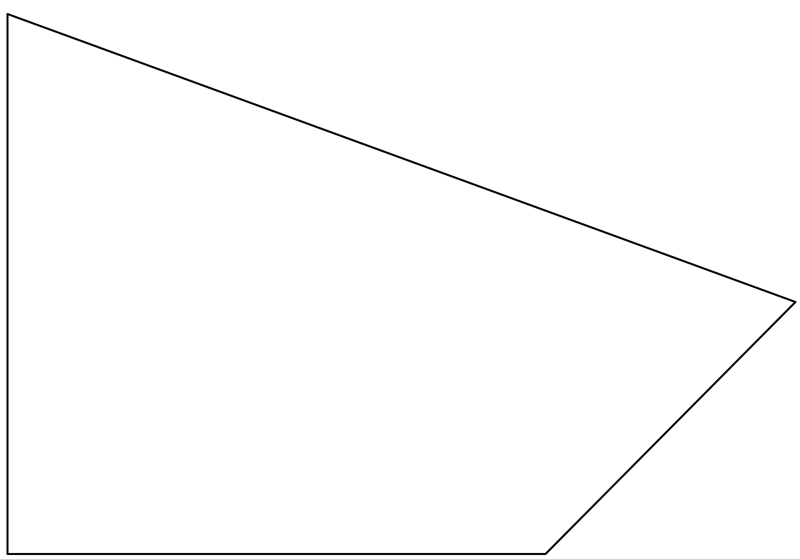




Quadrilateral Patterns

Page 3





Activity: Origami: Making a Square from a Non-Square Rectangle

Format: Individual/Small Group

Objectives: Participants will use non-square rectangular paper to construct a square through paper folding. They will also determine how many lines of symmetry in the square.

Related SOL: K.11, 1.16, 2.21, 3.18, 3.20

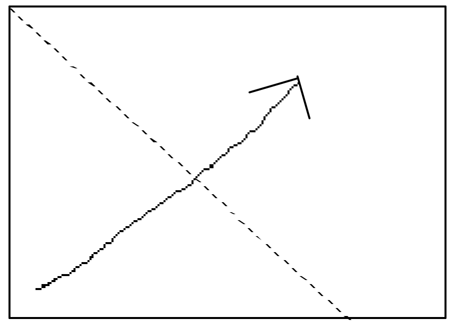
Materials: Scissors, non-square rectangular piece of paper, Folding a Square Activity Sheet

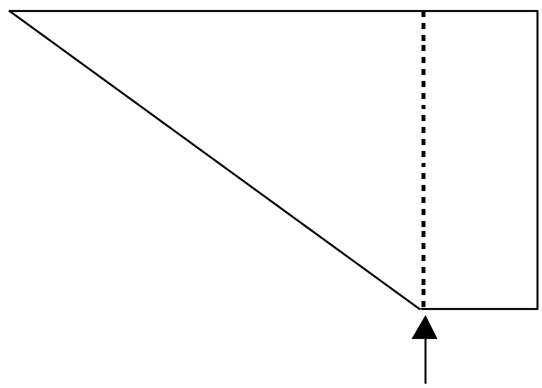
Time Required: Approximately 5 minutes

Directions:

1) Give each participant a pair of scissors and a non-square rectangular piece of paper. Have them fold the corner to the opposite side to form a triangle as shown on the Activity Sheet. Then have them cut along the vertical line to find as many ways as possible to fold the square in half so the halves match, i.e., find all the lines of symmetry.

Folding a Square





Activity: Origami: Making a Heart

Format: Individual/Small Group

Objectives: Participants will identify geometric figures and concepts related to symmetry used in paper folding.

Related SOL: K.11, 1.16, 2.21, 3.18, 3.20

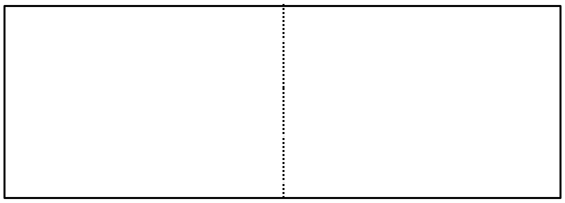
Materials: Piece of paper, 8½ inches by 2½ inches, and Making a Heart Activity Sheet

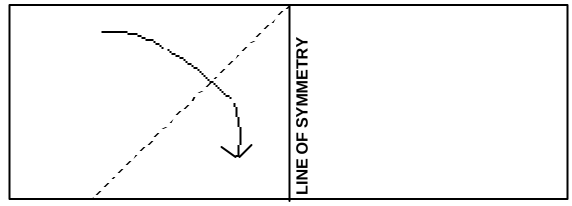
Time Required: Approximately 5 minutes

Directions:

1) Give each participant a piece of paper, 8½ inches by 2½ inches. Have them fold the heart as shown on the Activity Sheet.
2) At the first fold, point out the line of symmetry.
3) At step 4, discuss the pentagon formed.
4) At step 5, point out the isosceles right triangle formed.

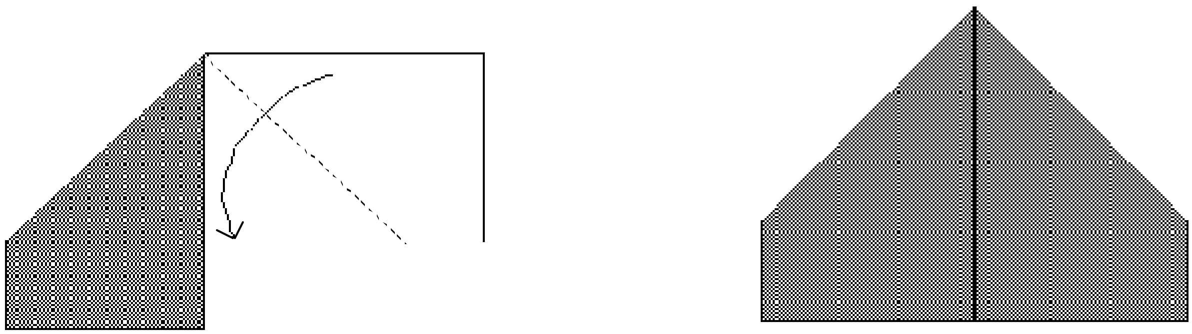
Origami: Making a Heart



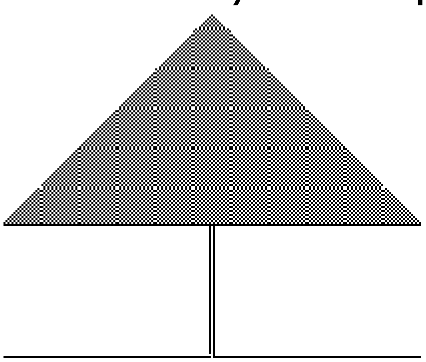


1) Fold the right edge to meet the left edge Unfold.

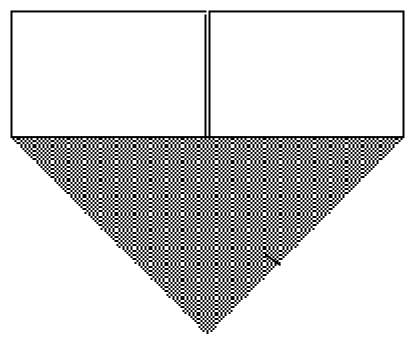
2) Fold the top left edge down to meet. the center crease (the line symmetry).



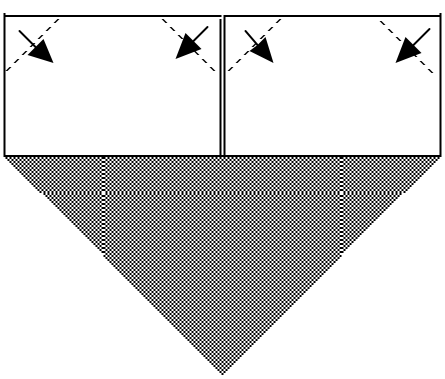
3) Fold the top right down to meet the center crease.



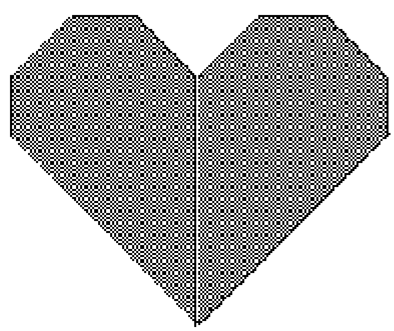
4) Turn over.



5) Now rotate 180.



6) Fold the four corners at the top down into small triangles.



7) Turn the heart over.

Elementary Geometry
Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Transformational Geometry: Tessellations	Sums of the Measures of Angles of Triangles	103	5.13		Paper, scissors, and rulers
	Do Congruent Triangles Tessellate?	106	3.18, 4.17, 5.13, 5.14	Types of Triangles, Sum of the Measures of the Angles, Tessellations of Triangles 1 and 2, Congruent Scalene Triangles	Paper, scissors, and rulers
	Do Congruent Quadrilaterals Tessellate?	113	4.17, 5.15	Sum of the Measures of the Angles, Tessellations of Quadrilaterals 1, 2, and 3	Paper, scissors, and rulers
	Tessellations by Translation	118	4.17, 5.15	Tessellations by Translation, Square	1 large square piece of paper per participant, rulers and scissors
	Tessellations by Rotation	121	4.17, 5.15	Tessellations by Rotation	1 large square piece of paper per participant, rulers and scissors
Solid Geometry	Solid Figure Sort	124	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	What's My Figure? Ask Me About It.	125	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	What's My Figure? Touch Me.	126	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	Take It Apart	127	2.20, 2.22, 3.18, 4.17, 5.16		Cardboard cereal boxes, canisters, milk cartons, scissors
	Building Solid Figures	128	2.20, 2.22, 3.18, 4.17, 5.16		Scissors and tape or glue; or solid figures, D-stix, or other commercial 3-dimensional building kit

Topic: Transformational Geometry: Tessellations

Description: Patterns of geometric design are all around us. We see them every day, woven into the fabric of the clothes we wear, laid underfoot in the hallways of the buildings where we work, and printed on the wallpaper of our homes. Whether simple or intricate, such patterns are intriguing to the eye. We will explore a special class of geometric patterns called *tessellations*. Our investigation will interweave concepts basic to art, to geometry, and to design.

The word *tessellation* means, "A design that completely covers the surface with a pattern of figures with no gaps and no overlapping." It comes to us from the Latin *tessela*, which was the small, square stone, or tile used in ancient Roman mosaics. *Tilings* and *mosaics* are common synonyms for tessellations. Much like a Roman mosaic, a *plane tessellation* is a pattern made up of one or more figures, completely covering a surface without any gaps or overlaps. Note that both two-dimensional and three-dimensional figures will tessellate. Two-dimensional figures may tessellate a plane surface, while three-dimensional figures may tessellate space. We will use the word *tessellation* alone to always mean a plane tessellation.

Although the mathematics of tiling can become quite complex, the beauty and order of tessellation is accessible to anyone who is interested. To analyze tessellating patterns, you have to understand a few things about geometric figures and their properties - but all you need to know is easily explained in a few pages.

We will approach this subject through directed exploration. We will be exploring the following questions: Which figures will tessellate (that is, tile a plane without overlapping or leaving spaces)? Why will certain figures tessellate and others not? How many different tessellating patterns can we create using two or more regular two-dimensional plane figures? Do tessellating designs have symmetry? If so, what kind? How can we use transformations (slides, flips, and turns) to create unique tessellations? What other techniques could we use to generate the intricate designs?

In the twentieth century, a number of fine artists have applied the concept of tessellating patterns in their work. The best known of these is Dutch artist M. C. Escher. Inspired by the Moorish mosaic designs he saw during a visit to the Alhambra in Spain in the 1930s, Escher spent most of his life creating tessellations in the medium of woodcuts. He altered geometric tessellating figures into such forms as birds, reptiles, fish and people.

Related SOL: K.11, K.12, 1.16, 2.21, 3.18, 3.20, 4.17, 5.13, 5.14

Activity: Sum of the Measures of Angles of a Triangle

Format: Large Group/Small group

Objective: Participants will demonstrate that the sum of the measures of the interior angles of any triangle equals 180°, by cutting three random triangles out of paper, numbering and tearing off their vertices, and rearranging them adjacent to each other to form a straight angle.

Related SOL: 5.13

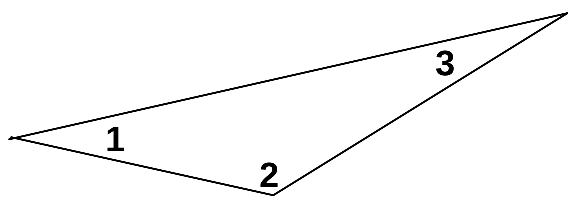
Materials: Paper, scissors, straightedges

Time required: Approximately 15 minutes

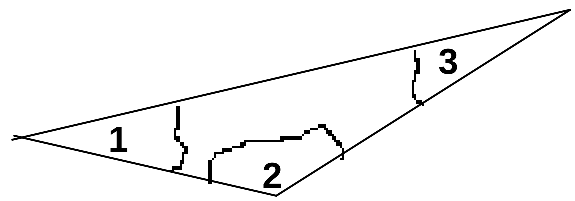
Directions:

1) Distribute the paper, scissors, and straightedges.

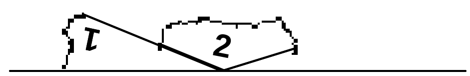
2) Have each participant draw 3 large triangles using a straightedge, and then cut the 3 triangles out. They should label the vertices of one triangle as 1, 2, and 3; label the vertices of the second triangles as 1*, 2*, and 3*; and label the vertices of the third triangle as 1', 2', and 3'.



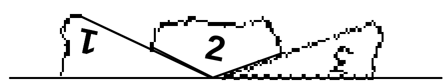
3. Have them tear off the corners. (That's right. Tear, not cut. By tearing, you can still determine which was the vertex. It will be the cut part.)



4. The participants should draw a dot on the page and a straight line through the dot. Have them place the cut vertex of <1 on the dot and the cut side on the line. They can trace the angle or tape it in place. They should choose another angle from the same figure and place its cut vertex on the dot, lining up the side of the first angle not on the line with a side of the second angle, and trace or tape it in place.



5. Have them repeat this sequence of steps until all the angles from one figure are adjacent to one another.



6. Ask what angle these combined corners form?

7. Repeat this procedure with the other two triangles. Ask if the same thing happens with all three triangles?

8. Have the groups share their results and then form conclusions focused on the following issues.
   a. Do the angles from all of their triangles always equal the same amount?
   b. Examine any triangle that turned out differently. Why do you think it did?
   c. Based on this activity, what conjecture can you make about the sum of the measure of angles in a triangle?

d. If you and your group members made 200 triangles, tore off the vertices, lined them up, and they all totaled the same, would this insure that the 201st triangle would come out the same?

e. If you could find a single triangle that came out differently, you would disprove the conjecture that the sum of the measures of the interior angles in a triangle is always 180°. Any item that doesn't fit your conjecture is called a **counterexample**. A single counterexample is enough to disprove a conjecture. Did you or any of your group members find a triangle whose angle measures didn't add up to 180°? Unfortunately, not finding one doesn't mean that one doesn't exist. However, it does give us more confidence in our conjecture.

Activity: Do Congruent Triangles Tessellate?

Format: Large Group/Small group

Objective: Participants will cut out sets of congruent triangles of various types and cover the plane with each type of triangle.

Related SOL: 3.18, 4.17, 5.13, 5.14

Materials: Paper, scissors, rulers, Types of Triangles Activity Sheet, Sum of the Measures of the Angles Activity Sheet, Tessellations of Triangles Activity Sheets 1 and 2, Congruent Scalene Triangles Activity Sheet

Time required: Approximately 20 minutes

Directions:

1) Remind the participants of the following definition: **Tessellation – a design that completely covers a surface with a pattern of figures with no gaps and no overlapping.**

2) Since a triangle is the simplest two-dimensional plane figure, we will start with the triangle in our investigation of which two-dimensional plane figures tessellate. Also, to keep things simple, have the participants explore tessellating with a single triangular figure rather than combinations of different triangular figures.

3) Review the definition of congruent triangles -- triangles that have the same size and figure.

4) Before proceeding with the investigation, have the participants look at some different types of triangles so that we can consider each type separately. Triangles are classified according to the relationships and the size of their sides and angles. Examples of each triangle type are shown on the Types of Triangles Activity Sheet.

5) Ask the participants to explore the following questions:

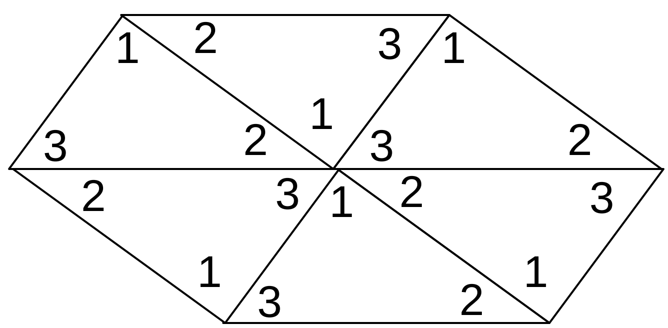
Which triangles (if any) tessellate?

If some triangles tessellate, do they all?

Start the investigation with scalene triangles. Have the participants draw a scalene triangle and copy it several times to make congruent triangles. They should label them so that all angle 1's are congruent, all angle 2's are congruent, and all angle 3's are congruent. The participants should cut the congruent triangles out and move them around to see if they tessellate. If you don't want the participants to create their own scalene triangles, you can just have them cut out several triangles from the Congruent Scalene Triangles Activity Sheet

6) Ask the participants if angles 1, 2, and 3 come together at a common vertex? What appears to be the sum of the measures of triangles 1, 2, and 3?

7) Using six congruent scalene triangles, we can completely fill all the space around the common vertex point of the six triangles. There are no gaps, no overlaps - a criterion of a tessellation. Do you think that all scalene triangles will tessellate in the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.



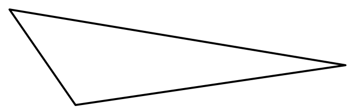
8) Try to make a tessellation using six congruent right triangles. Do you think that all right triangles will tessellate in the plane? Why or why not?

9) Can you make a tessellation using six congruent equilateral triangles or six congruent isosceles triangles?

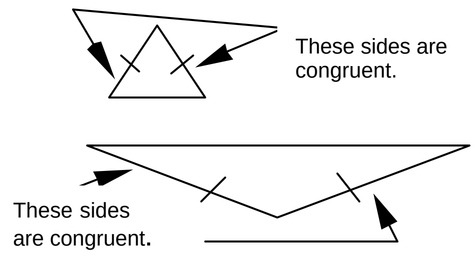
10) Examine the tessellations on the Tessellations of Triangles Activity Sheet. Identify the triangles used to form the tessellations. Do you think that any six congruent triangles can be used to make a tessellation? Why or why not?

Types of Triangles

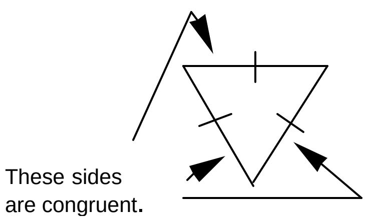
A **scalene** triangle has **no** congruent sides.



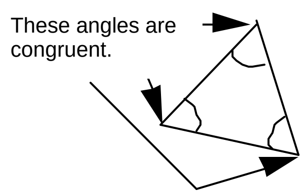
An **isosceles** triangle has **two or more sides**



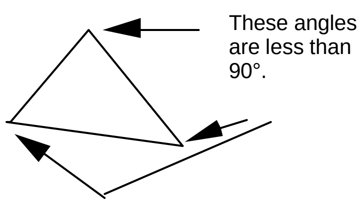
An **equilateral** triangle has all three sides congruent.



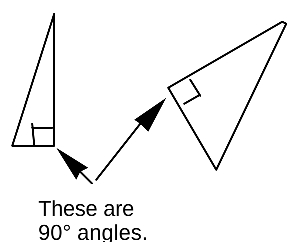
An **equiangular** triangle has all three angles congruent.



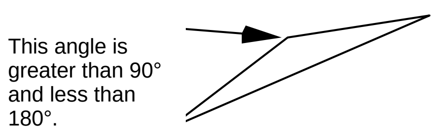
An **acute** triangle has all three angles acute.



A **right** triangle contains a right angle.

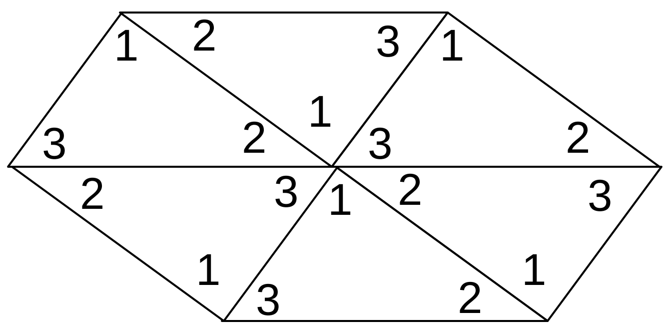


An **obtuse** triangle contains an obtuse angle.



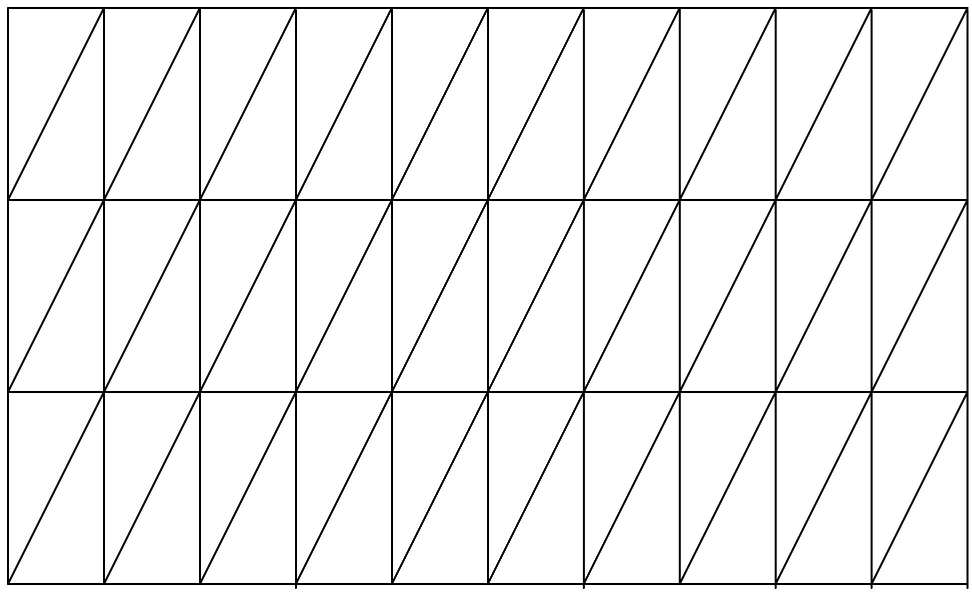
Sum of the Measures of the Angles

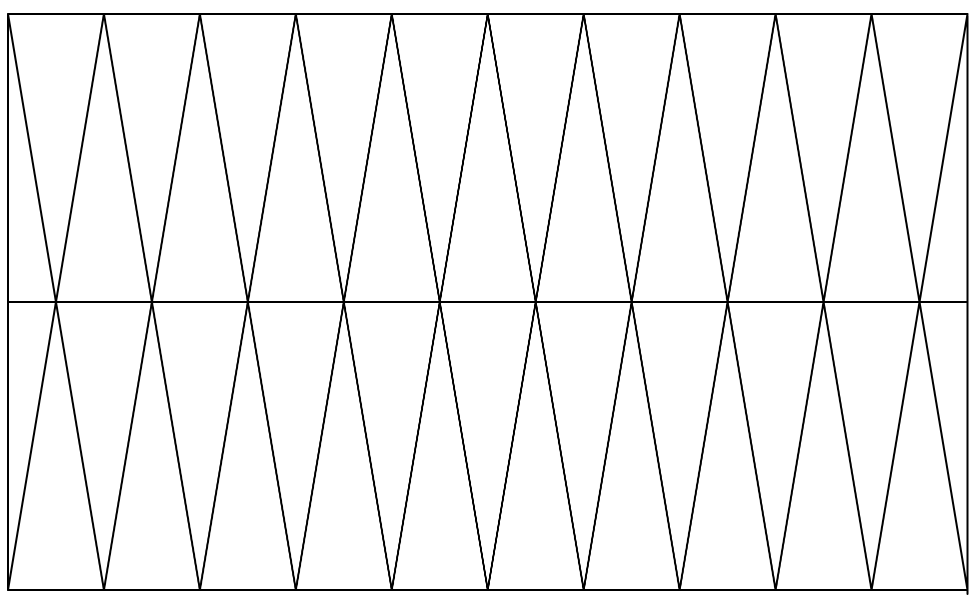
What is the sum of the measures of the angles about the center point?



Tessellations of Triangles

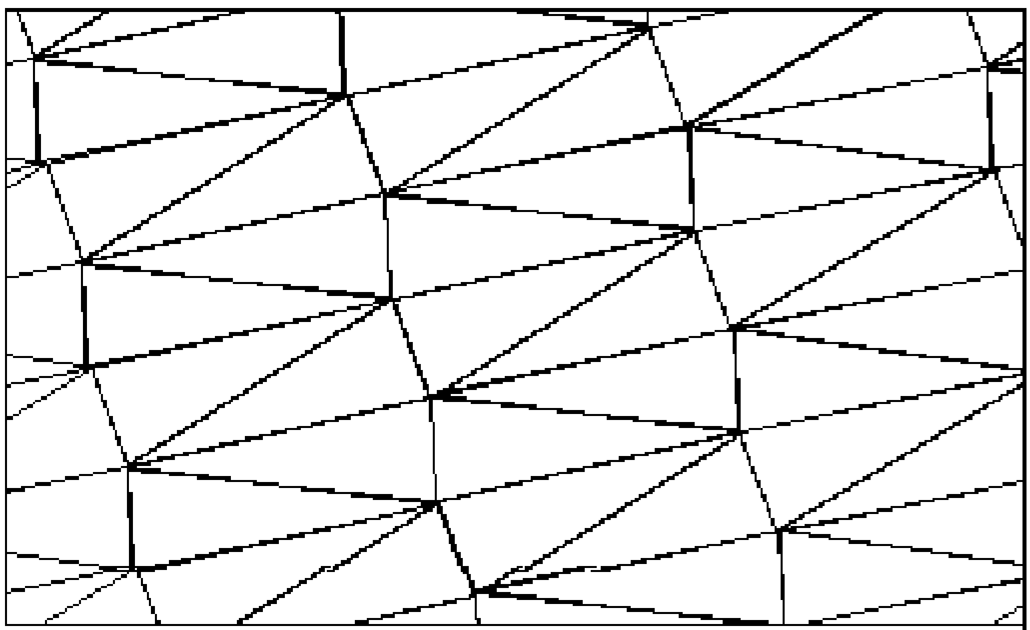
Page 1

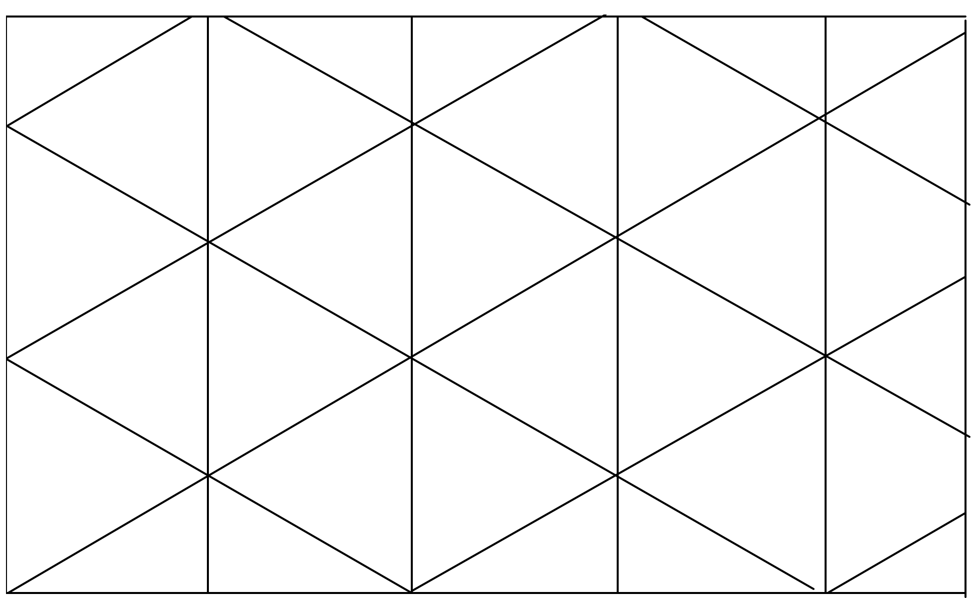




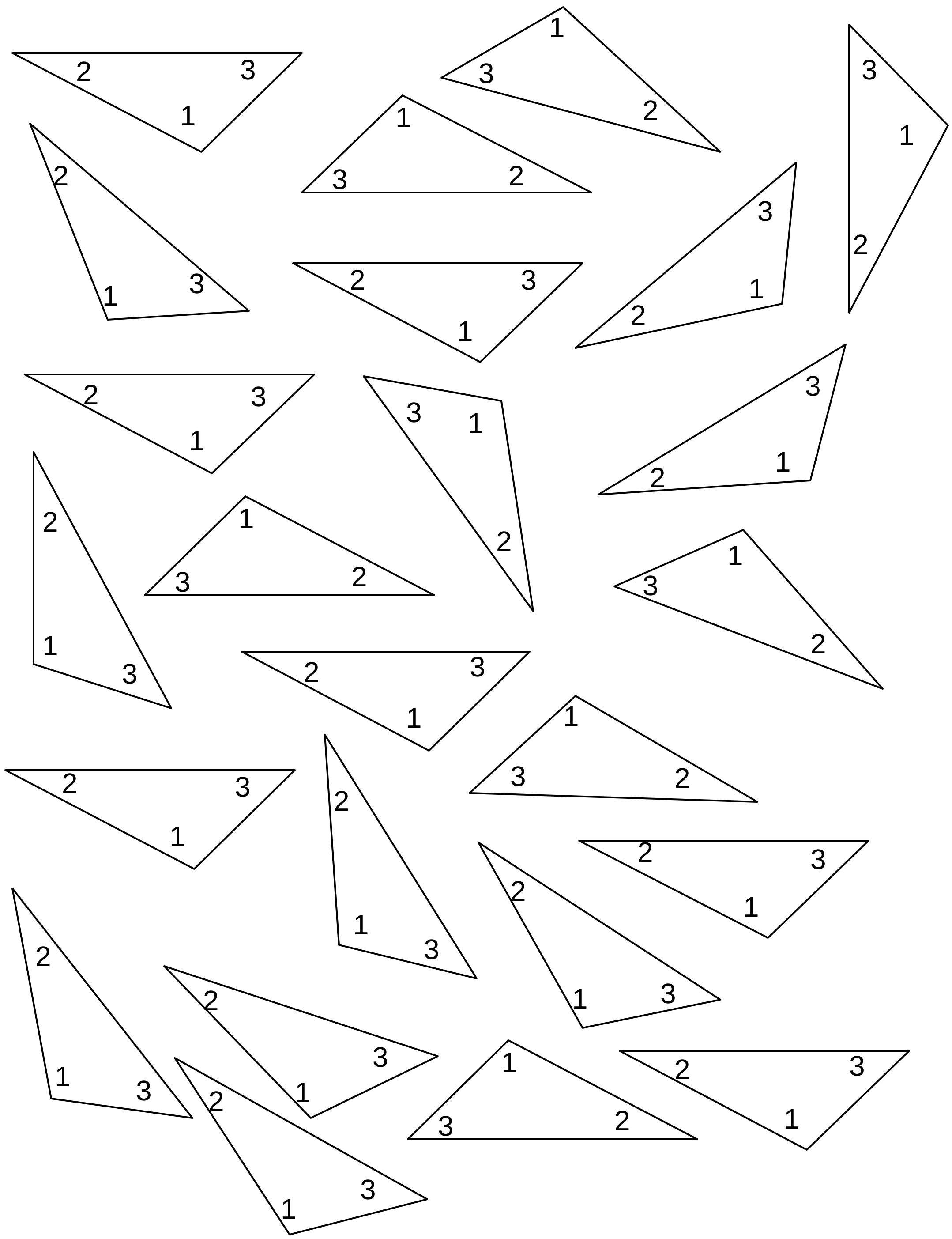
Tessellations of Triangles

Page 2





Congruent Scalene Triangles



Activity: Do Congruent Quadrilaterals Tessellate?

Format: Large Group/Small Group

Objective: Participants will determine whether congruent quadrilaterals tessellate.

Related SOL: 4.17, 5.15

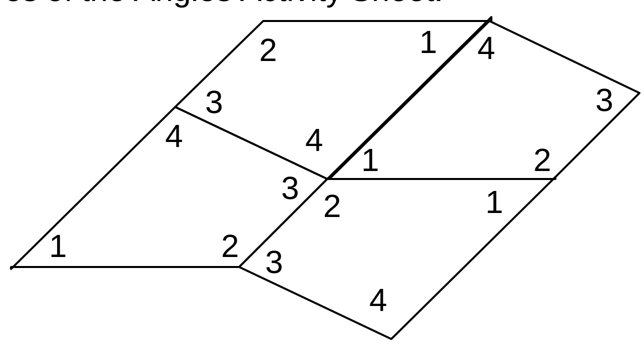
Materials: Paper, scissors, rulers, Sum of the Measures of the Angles Activity Sheet, Tessellations of Quadrilaterals Activity Sheets 1, 2, and 3

Time required: Approximately 15 minutes

Directions:

1) Repeat the previous activity entitled "Do Congruent Triangles Tessellate?" (page 106), using six or more congruent quadrilaterals (four-sided two-dimensional plane figures). Do your quadrilaterals tessellate?

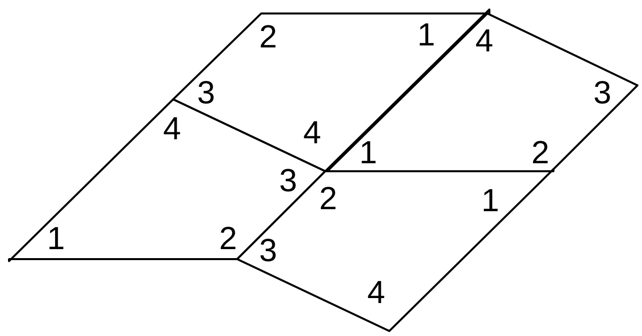
2) Using four congruent quadrilaterals, we can completely fill all the space around the common vertex point of the four quadrilaterals. There are no gaps, no overlaps - a criterion of a tessellation. Do you think that all quadrilaterals will tessellate in the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.



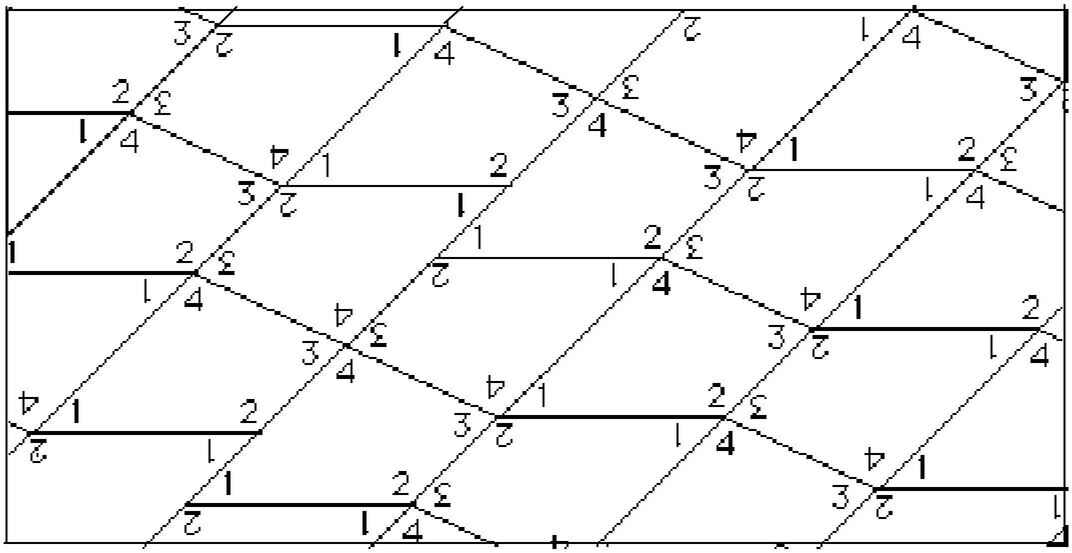
3) Examine the tessellations in the Tessellations of Quadrilaterals Activity Sheets. Identify the quadrilaterals used to form the tessellations. Do you think that any four congruent quadrilaterals can be used to make a tessellation? Why or why not?

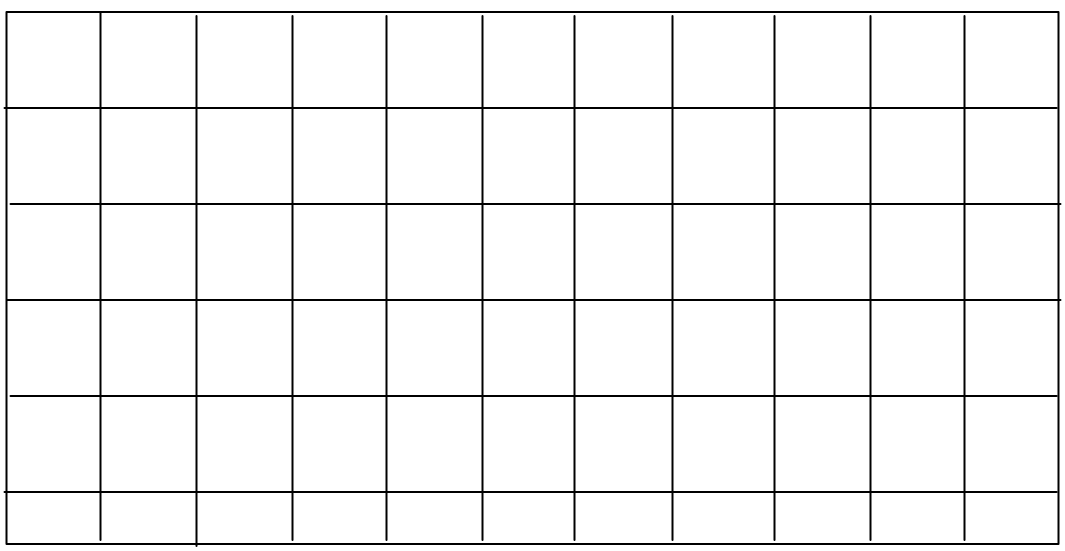
Sum of the Measures of the Angles

What is the sum of the measures of the angles about the center point?



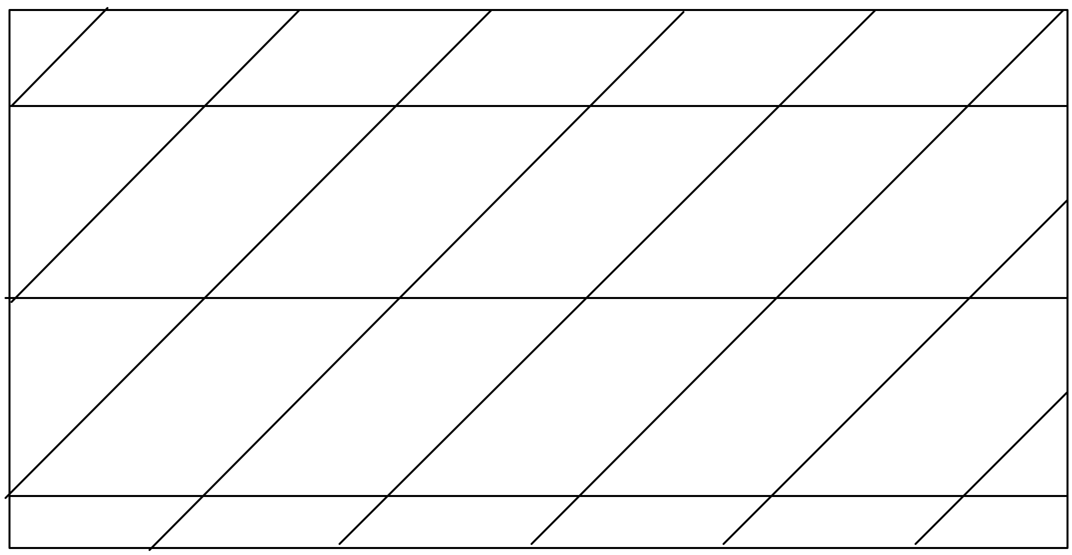
Tessellations of Quadrilaterals
Page 1

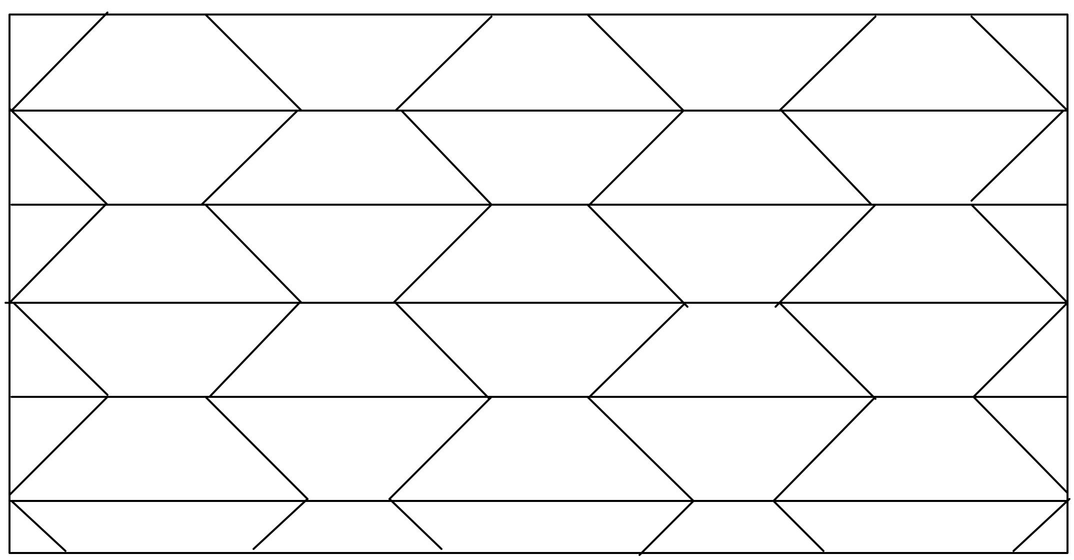




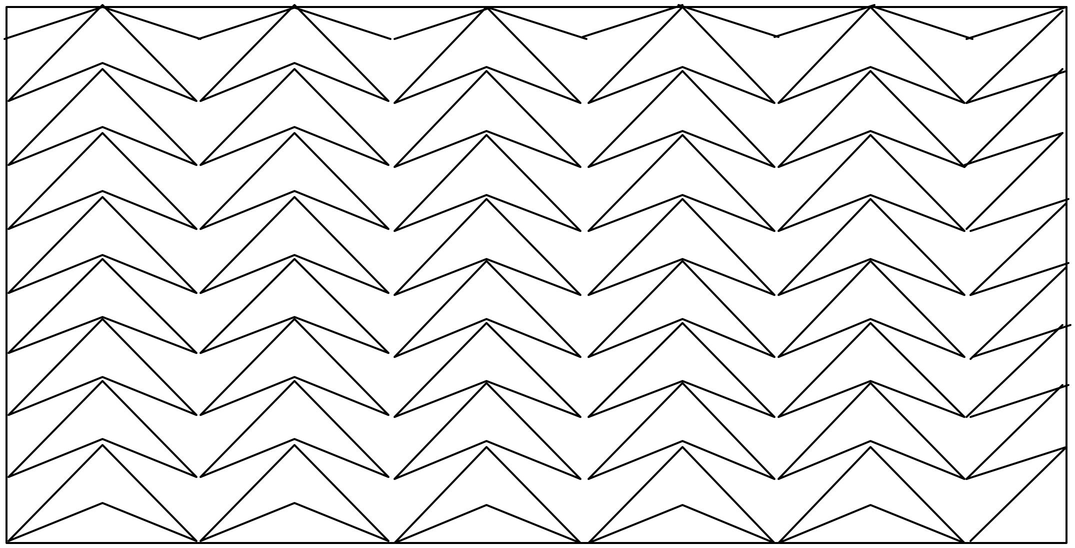
Tessellations of Quadrilaterals

Page 2





Tessellation of Quadrilaterals
Page 3



Activity: Tessellations by Translation

Format: Individual

Objective: Participants will create their own tessellations using translations.

Related SOL: 4.17, 5.15

Materials: One large square piece of paper, straightedge, scissors, Tessellations by Translation Activity Sheet, Square Activity Sheet

Time required: Approximately 40 minutes

Directions:

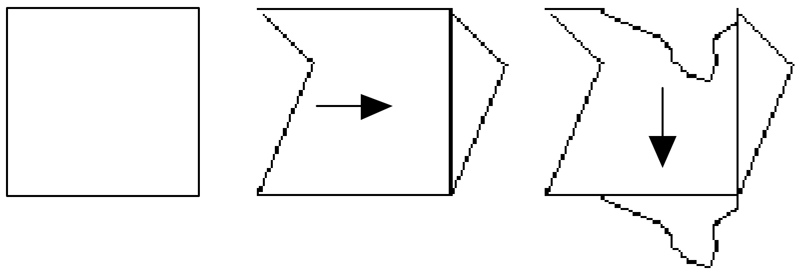
1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and slide it across to the other side. Trace it as shown on the Tessellations by Translation Activity Sheet.

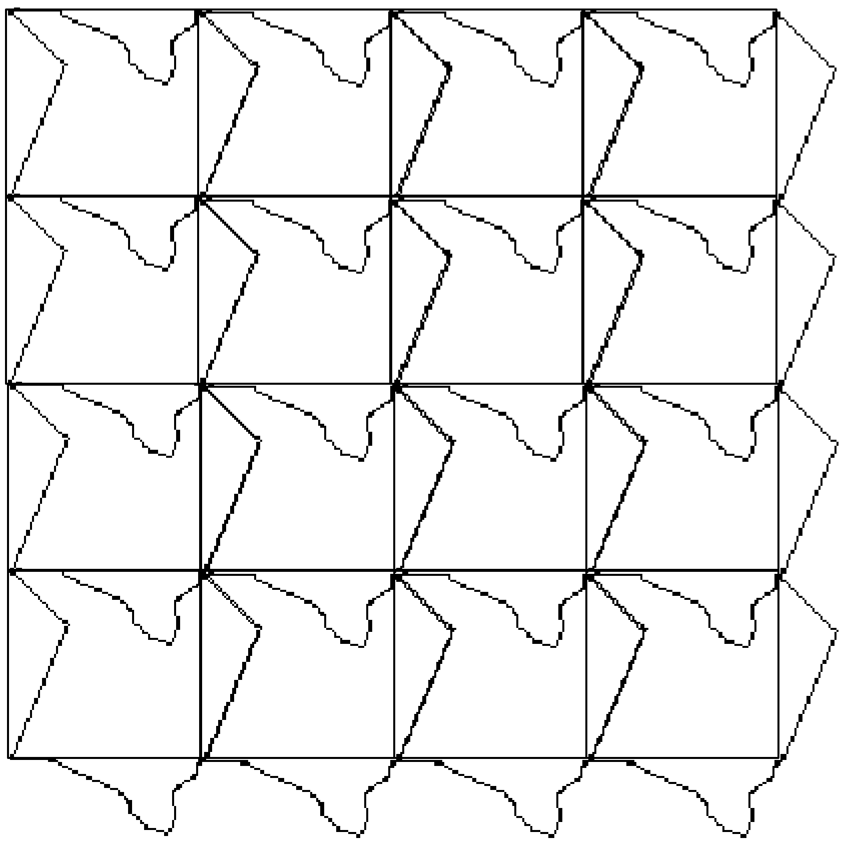
2) Sketch a figure extending into the square from the top on Handout 4.3. Cut out that figure from the top of the square and slide it down to the bottom. Trace it as shown on Tessellations by Translation Activity Sheet.

3) Cut out the other side and the bottom of the square along the figures you traced. Now you have your pattern to tessellate.

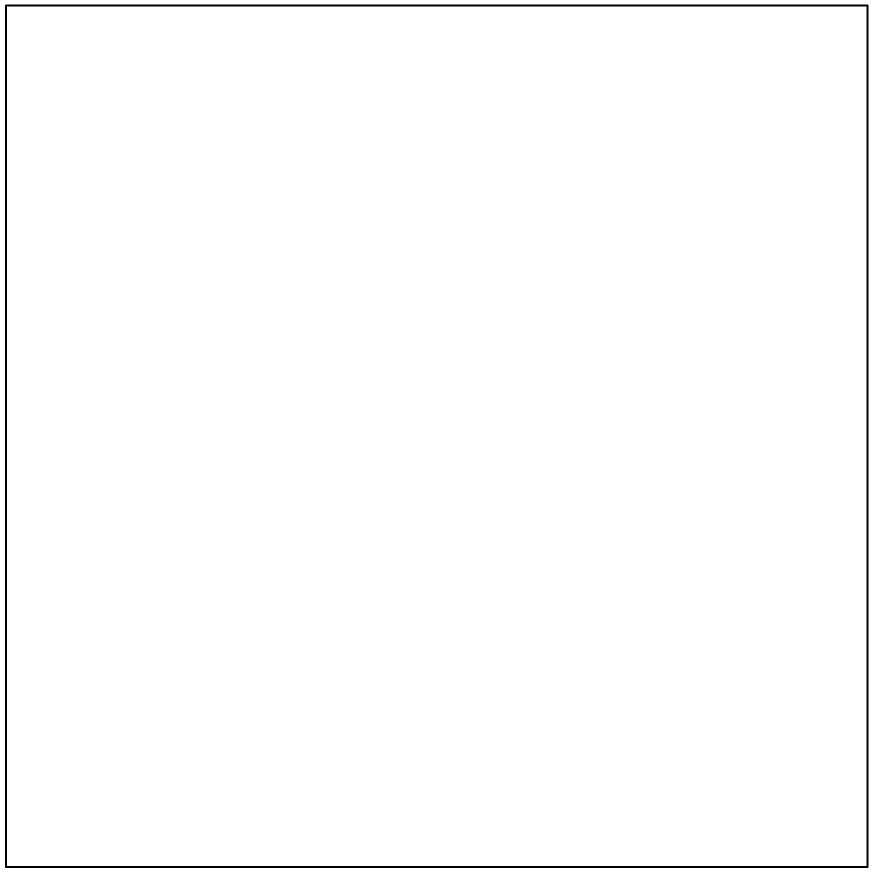
4) Trace the pattern repeatedly, translating it as shown on Tessellations by Translation Activity Sheet to form a tessellation.

Tessellations by Translation





Square



Activity: Tessellations by Rotation

Format: Individual

Objective: Participants will create their own tessellations using rotations.

Related SOL: 4.17, 5.15

Materials: One large square piece of paper, scissors, Tessellations by Rotation Activity Sheet, Square Activity Sheet (from previous activity)

Time required: Approximately 40 minutes

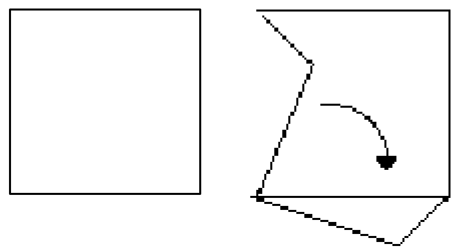
Directions:

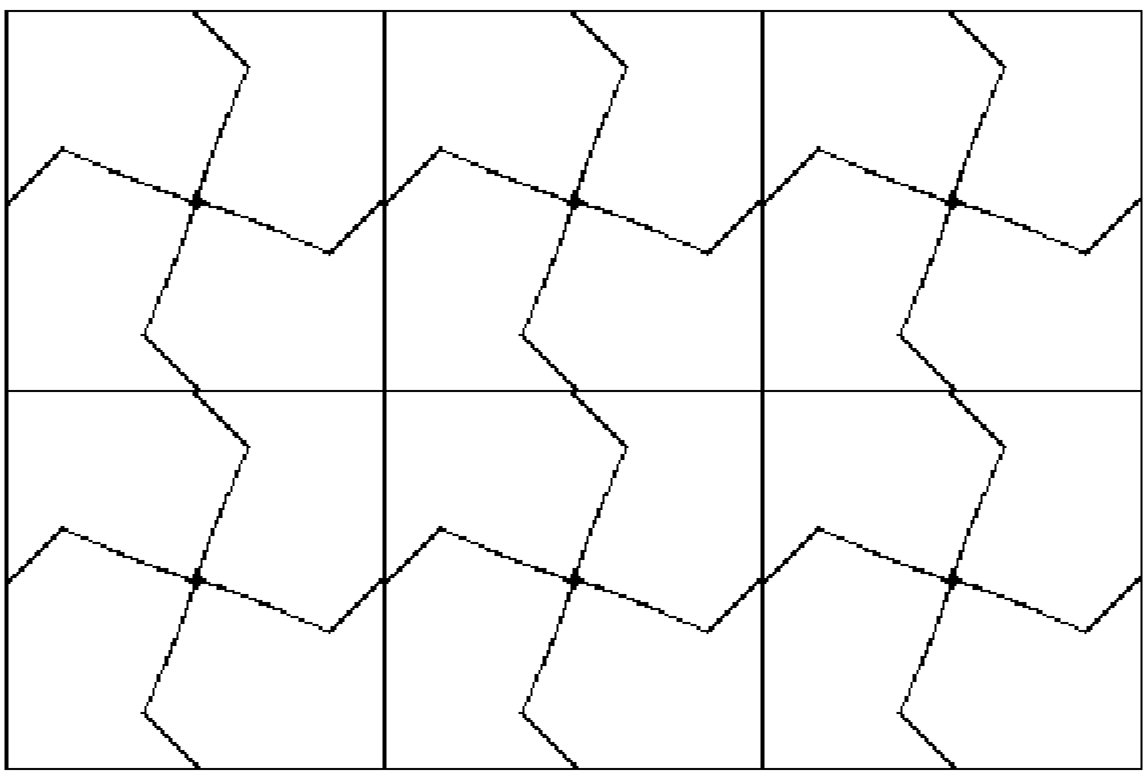
1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and rotate it across to the other side. Trace it as shown on Tessellations by Rotation Activity Sheet.

2) Cut out the figure you traced and the other sides of the square. Now you have your pattern to tessellate.

3) Trace the pattern repeatedly, translating it as shown on Tessellations by Rotation Activity Sheet to form a tessellation.

Tessellations by Rotation





Topic: Solid Geometry

Description: Participants will explore three-dimensional figures by sorting and classifying them, determining what they are by touch, building them, and taking them apart.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Activity: Solid Figure Sort

Format: Large Group/Small Group

Objectives: After performing their own sorts, participants will be able to distinguish the way students at various van Hiele levels of geometric understanding may sort solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids per group of 4-6

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into small groups. Distribute the sets of geometric solids, one set per group of 4-6 participants. Have them sort the solids into groups that belong together, sketching the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

2) Ask the participants to describe their sorts. Have them compare their sorts with those of other groups.

Activity: What's My Figure? Ask Me About It.

Format: Large Group

Objectives: Participants will use logical reasoning to determine the solid figure in the box after asking questions and receiving information about it.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids, box or bag

Time Required: Approximately 10 minutes

Directions:

1) The trainer says to the participants, "This box (or bag) contains a solid figure." Shake it so the participants can hear. "I'd like you to ask me some questions that can be answered with yes or no to figure out what is in the box."

2) Questions and answers continue until the participants can figure out what solid figure is in the box.

Activity: What's My Figure? Touch Me.

Format: Large Group

Objectives: Participants will determine the solid figure in the bag by touch alone.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids, a paper bag

Time Required: Approximately 10 minutes

Directions:

1) The trainer says to the participants, "This bag contains a solid figure." Shake it so the participants can hear. "One of you at a time may put your hand into the bag and touch the solid. Try to figure out what is in the bag."

2) The participants take turns touching the solid figure in the bag without looking and try to determine the solid figure.

Activity: Take It Apart

Format: Large Group/Small Group

Objectives: Participants will determine the figures that form various solid figures and analyze how they fit together to form the solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: Cardboard cereal boxes, canisters, milk cartons, etc., emptied and cleaned, scissors

Time Required: Approximately 10 minutes

Directions:

1) The trainer or group collects a variety of cardboard containers such as cereal boxes. The participants carefully cut apart containers along their seams, in such a way that they can be flattened out, but each piece is connected. This figure is called the **net**.

2) The group should identify each figure formed.

3) The group should see how many different nets they can find for the same container.

Activity: Building Solid Figures

Format: Large Group/Small Group

Objectives: Participants will determine the plane figures that form various solid figures and analyze how they fit together to form the solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: Scissors, tape or glue, geometric solids, and handout made by tracing the sides of various geometric solids; or Polydrons, D-stix, or other commercial three-dimensional building kit

Time Required: Approximately 20 minutes

Directions:

1) Choose a geometric solid from the set. Make a handout by tracing each of the faces to form its net. Repeat for two or three more solids.

2) Distribute scissors, tape or glue, and multiple copies of handout; or Polydrons, D-stix, or other commercial three-dimensional building kit.

2) Working in small groups, the participants should predict which solid the net will make. The trainer may show several solids, one of which is the one that the net will form.

3) After the predictions are made, the participants should cut out the net and tape it together.

4) The participants should compare their solids to their predictions and to the original solids.

Elementary Geometry
Session 5

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Perimeter And Area, Part I	Dominoes and Triominoes	131	K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15	One-Inch Grid Paper	Tiles, scissors
	Tetrominoes	134	K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15	One-Inch Grid Paper, Sorting Mat	Tiles, scissors, paste (optional)
	Pentominoes	137	K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15	Pentomino Alphabet, Solutions Sheet	Tiles, scissors, and envelopes or plastic baggies
	Areas with Pentominoes	141	K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4,13, 5.10	Pentomino Puzzles, 3 x 5 Solutions, 4 x 5 Solutions, Patterns for Pentominoes 1 and 2	Scissors and envelopes or plastic baggies; or sets of Pentominoes
	Hexominoes	148	K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15	Hexominoes 1, 2, and 3	Tiles, scissors, grid paper, and envelopes
	Perimeters with Hexominoes	152	K.11, K.12, K.13, 1.16, 2.20, 3.18, 3.20, 4.13, 5.10	Perimeters of Hexominoes	Hexominoes from previous activity
Perimeter And Area, Part II	The Perimeter is 24 Inches. What's the Area?	155	4.13, 5.8, 5.10	One-Inch Grid Paper, One-Half-Inch Grid Paper	24-inch paper strip (collar), 38 one-inch cubes, grid paper
	The Area is 24 Inches. What's the Perimeter?	158	4.13, 5.8, 5.10		24 one-inch squares and grid paper
	Change the Area	159	4.13, 5.8, 5.10	Geoboard Dot Paper	Square geoboard, overhead Geoboard, and rubber bands
Coordinate Geometry	Hurkle	162	4.18	Find The Hidden Hurkle 1, 2, and 3	
	Two-Dimensional Hurkle	166	4.18	Find The Hidden Hurkle Coordinate Grid	

Topic: Perimeter and Area, Part I

Description: Participants will explore the concept of area through the use of "-ominoes" formed by arranging sets of squares. Putting two squares together, with full edges touching forms the domino, the most familiar "-ominoes." Using the "-ominoes" rule (see below), there is only one figure that you can make with two squares. However, by adding squares, other "-ominoes" can be produced. Working with "-ominoes," the participants will have the opportunity to:

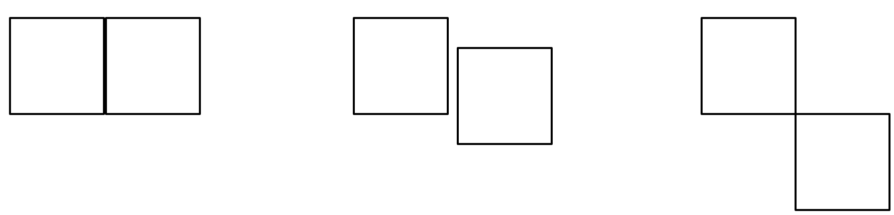
- use spatial visualization to build figures
- develop strategies for finding new figures
- test to find figures that are flips and turns of other figures
- reinforce their understanding of congruence by flipping and turning figures in order to compare them
- use "-ominoes" to fill given rectangular areas
- sort "-ominoes" according to their perimeter and look for patterns in their collected data.

A unit on "-ominoes" can be developed through the use of materials such as color tiles, paper squares, the orange block in the pattern block set, interlocking cubes, or grid paper. Most "-ominoes" activities are best done where children work in groups of two to four, each group with its own set of materials. In that format, groups can explore the various problems independently. All activities should be conducted in an easy-going manner that encourages risks, good thinking, attentiveness and discussion of ideas.

"-Ominoes" Rule:

At least one full side of each tile must touch one full side of another tile.

Touching | Not Touching | Not Touching



Related SOL: K.11, K.12, K.13, 1.16, 2.20, 2.22, 3.18, 3.20, 4.13, 4.17, 5.10, 5.15

Activity: Dominoes and Triominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "-ominoes" with two and three tiles. They will test to find figures that are the result of flips and turns.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15

Materials: Tiles, scissors, grid paper preferably with the same size squares as the tiles, One-Inch Grid Paper Activity Sheet

Time Required: Approximately 20 minutes

Directions:

1) Show the participants a **domino** and ask them to describe everything they can about the domino. Putting two squares together, with full edges touching forms this figure. There is only one figure that you can make with two squares. Establish that it's made up of two tiles and that each tile must have one side in common with the other tile.

2) Ask the participants if they can find another way to arrange the two tiles so that every tile touches at least one complete side of another tile. They may describe the 90-degree rotation as different. As they discuss their strategies for determining whether or not two figures are the same, you may want to informally introduce the terms *flip* (reflect) and *turn* (rotate) used in transformational geometry.

3) Ask the participants to arrange three tiles in as many ways as possible and to tell something about the figure. Establish that it is made up of three color tiles and that every tile shares at least one complete side with another tile. Tell them that this is called a **triomino**, noting that the "tri" stands for three like in triangle.

4) Ask the participants to find another way to arrange the three tiles so that every tile touches at least one complete side of another tile. Have volunteers who think they have found different figures display them.

5) Model how to record the first figure on grid paper and cut it out. Then have the participants record and cut out their figures. Collect the cutout figures, making two piles, one for each of the two possible arrangements. Show how, by flipping (reflecting) or turning (rotating) the pieces in each pile, you can fit them over each other exactly to make a neat stack.

One-Inch Grid Paper

Activity: Tetrominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "tetrominoes" with four tiles, testing for figures that are flips and/or turns of other figures.

Related SOL K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15

Materials: Tiles, scissors, paste (optional), One-Inch Grid Paper from previous activity, Sorting Mat Activity Sheet

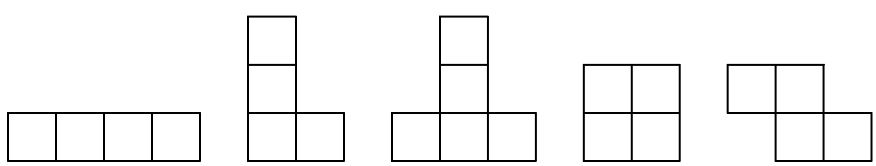
Time Required: Approximately 15 minutes

Directions:

1) Ask the participants to work in pairs. Distribute at least 30 color tiles per pair, scissors, Grid Paper and Sorting Mat Activity Sheets. Ask each participant to make a figure with four color tiles that follows the rule each square tile has one side in common with another square tile. These figures are called **tetrominoes**.

2) Ask the partners to compare their figures. If both made the same figure, have them copy it only once onto the grid paper and cut it out. If they made two different figures, have them copy and cut out both figures. Ask the participants how many different figures they think they can make with four color tiles.

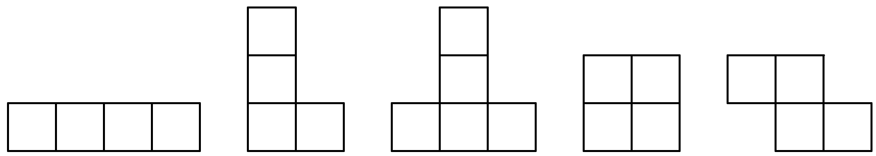
3) Ask the participants to make more figures with four color tiles. Each time they should compare the figures and record and cut out each different figure. The partners should continue until neither can think of any new figures to make and then count all their different figures, recording that number.



4) To promote thinking and sharing, use prompts such as these to promote class discussion:

   - How did you and your partner decide whether or not your figures were different? (Be sure to discuss flips [reflections], turns [rotations], and congruency here.)
   - Did you use any of your old figures to find new ones? If so, how did you do this?
   - How did you know that you found all of the different figures?
   - How many different figures are there? (5)

5) Invite participants to determine which of the tetrominoes are regular and which are irregular. (**Regular tetrominoes** are those that are squares or rectangles, while irregulars are not.)



Regular Irregular Irregular Regular Irregular

This is another way to reinforce the attributes of squares and rectangles. You might contrast this use of the "regular" with its more general meaning. A **regular polygon** is a polygon that has all congruent sides and all congruent angles. So a rectangle is a regular tetromino, but not a regular polygon. A square is both a regular tetromino and a regular polygon. With teachers, be sure to point out that the definition of regular tetromino is actually redundant because since a square is a type of rectangle, it would be enough to say that a regular tetromino is a rectangle. (Ask at what level this distinction should be discussed with students.) Invite participants to come to the front of the class and place their tetrominoes into one of the categories on the Sorting Mat Activity Sheet and give the reason for their decision.

6) Have participants trace or glue their tetrominoes into the appropriate columns on the Sorting Mat Activity Sheet.

Sorting Tetrominoes

Sort your figures into regular tetrominoes and irregular tetrominoes. If an arrangement of squares does not form a rectangle or a square, then it is irregular. Trace or paste the tetrominoes into the proper column below.

Regular Tetrominoes	Irregular Tetrominoes

Activity: Pentominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "pentominoes" with five tiles, testing for figures that are flips and turns of other figures.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15

Materials: Tiles, scissors, envelopes or plastic baggies, One-Inch Grid Paper Activity Sheet (used in Domino and Triominoes Activity), Pentomino Alphabet Activity Sheet, Solutions Sheet

Time Required: Approximately 15 minutes

Directions:

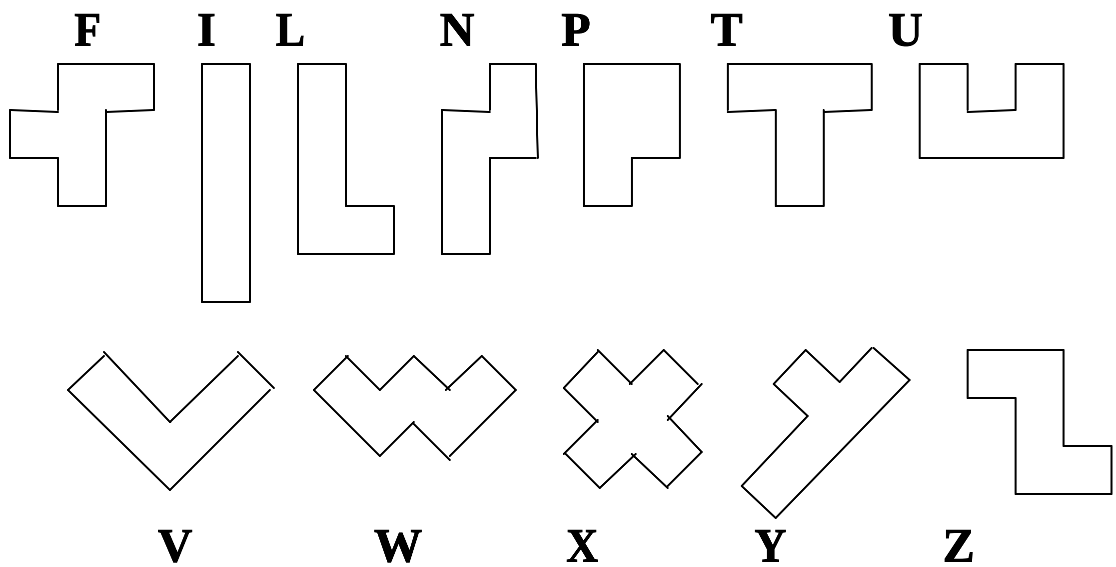
1) Distribute the materials and ask the participants to:
   a) Use five color tiles to find all the possible pentominoes. A **pentomino** is a figure made of five squares, each of which has one side in common with another of the squares.
   b) With a partner, use five color tiles of one color to make a pentomino. Record the pentomino on grid paper. Then make a different pentomino. Continue making and recording pentominoes until you cannot make any more that are different.
   c) Cut out the pentominoes. Compare them by flipping (reflecting) and turning (rotating) them to see if any match exactly. If you find a match, keep only one of them.
   d) Number your pentominoes and put them in an envelope. Write the number on the envelope. Exchange envelopes with another pair of participants. Check their pentominoes to see that none of them are the same. Mark any that you think are exactly the same. Return the envelopes. Check your envelope to see if any duplicates were found.
   e) Discuss ways you could sort your pentominoes.

2) Ask one or two pairs to post their pentominoes in an organized way. Note the organizations, and then discuss each of the posted pentominoes. Point out any duplicates and figures that are not pentominoes. Call on volunteers to supply any missing pentominoes.
   Use prompts such as these to promote class discussion:
   - Do you think that you have found all possible pentominoes? Explain.
   - In what ways do the arrangements differ from one another?
   - What strategies did you use to make new pentominoes from your old ones?
   - Did you find any patterns while making your pentominoes? Did you use these patterns when sorting them? How?
   - Did you sort the pentominoes according to the letters of the alphabet that they resemble? (Use Pentomino Alphabet Sheet to illustrate.)
   - Did sorting your pentominoes help you find others that were missing? If so, explain.

3) Ask the participants to predict which figures can be folded to make topless boxes. Have them put their figures into two groups: those that can be folded into boxes and those that cannot.

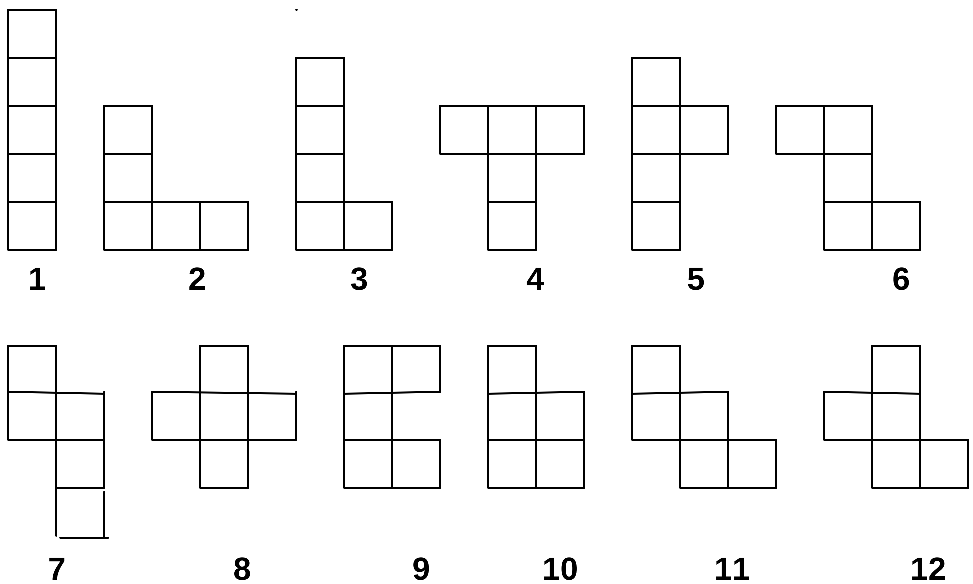
4) Have the participants fold their figures to test their predictions. Share the Solutions Sheet with participants.

The Pentomino "Alphabet"



Solutions Sheet

Which pentominoes fold up into an open top box?



Figures 3, 4, 5, 6, 7, 8, 11, and 12 will fold into an open top box.

Activity: Areas with Pentominoes

Format: Large Group/Small Group

Objectives: Participants will use "-ominoes" to fill given rectangular areas.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4,13, 5.10

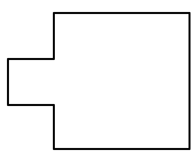
Materials: Scissors, envelopes or plastic baggies or sets of pentominoes made in previous activity, Pentomino Puzzles Activity Sheet, 3 x 5 Solutions Sheet, 4 x 5 Solutions Sheet, Patterns for Pentominoes Sheets 1, 2, and 3

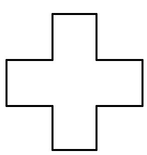
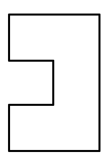
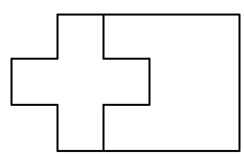
Time Required: Approximately 25 minutes

Directions:

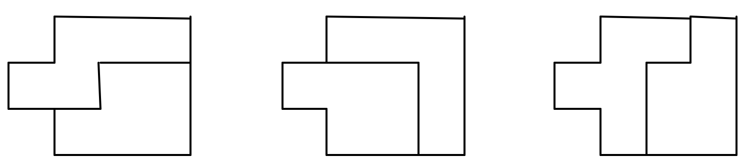
1) Ask the participants to use the pentominoes from the previous activity or have them carefully cut out a set from the Patterns from Pentominoes Activity Sheet.

2) Invite the participants to take their pentominoes and fit them together like jigsaw puzzle pieces. Point out that there are usually several ways to combine pentominoes to fill a given area.

For example,  can be made by combining

 and  like this 

or in several other ways.



3) Distribute the Pentomino Puzzles Activity Sheet and have the participants fill in the 3 x 5 and 4 x 5 areas with pentominoes. Demonstrate an example with the pentominoes. Ask the participants to keep a record of their work on a separate piece of grid paper. Have them compare their results with the Solution Sheets.

4) Ask the participants how many square units there are in each pentomino (five). Ask them to find the pentomino with the largest perimeter and the pentomino with the smallest perimeter. Discuss how figures with the same area can have different perimeters.

5) Ask participants how many square units there are in all 12 pentominoes combined (60). Have them design a symmetrical figure with an area equal to 60 square units and then see if it can be completely filled in with all twelve pentominoes. Share results.

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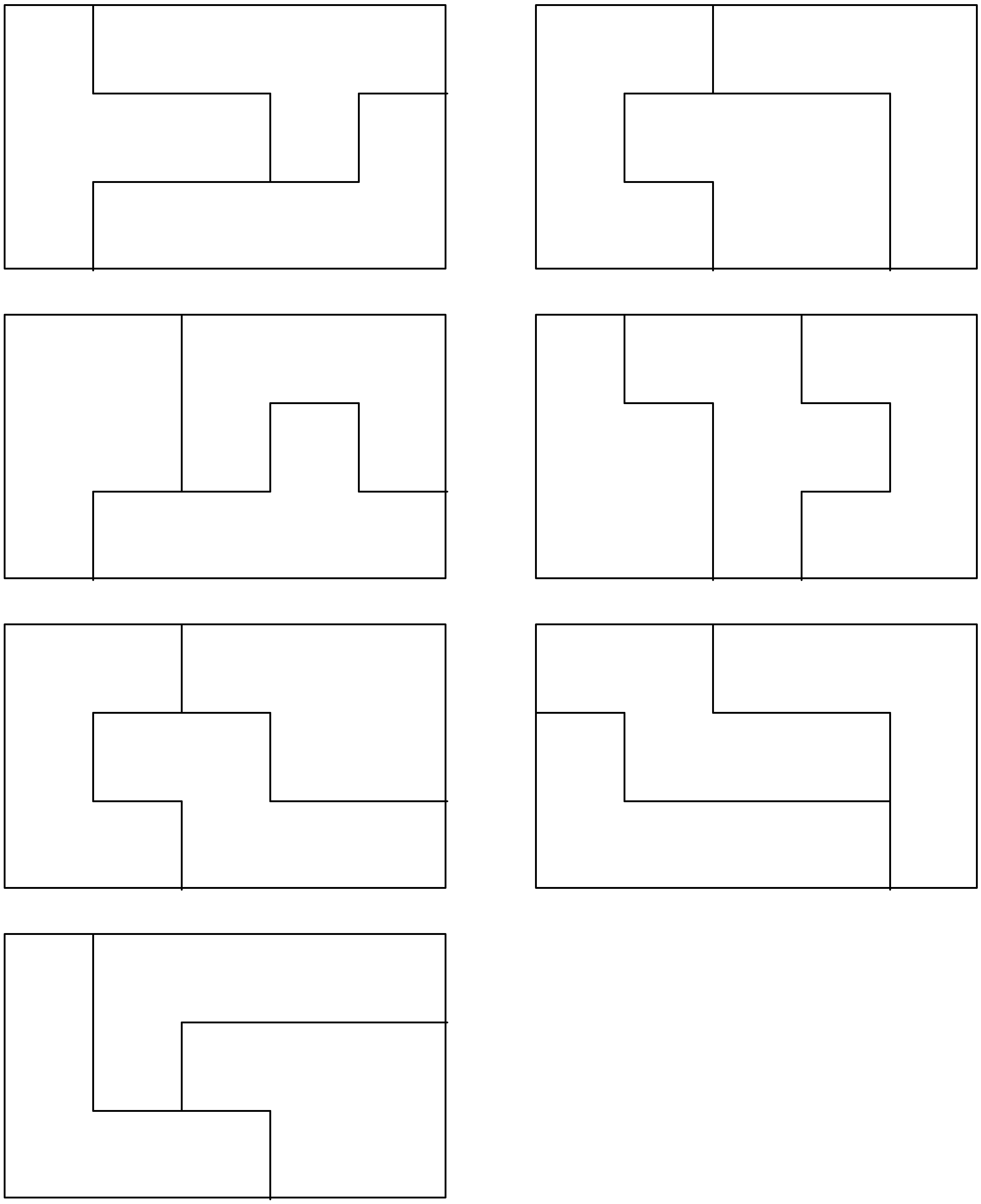
Pentomino Puzzles

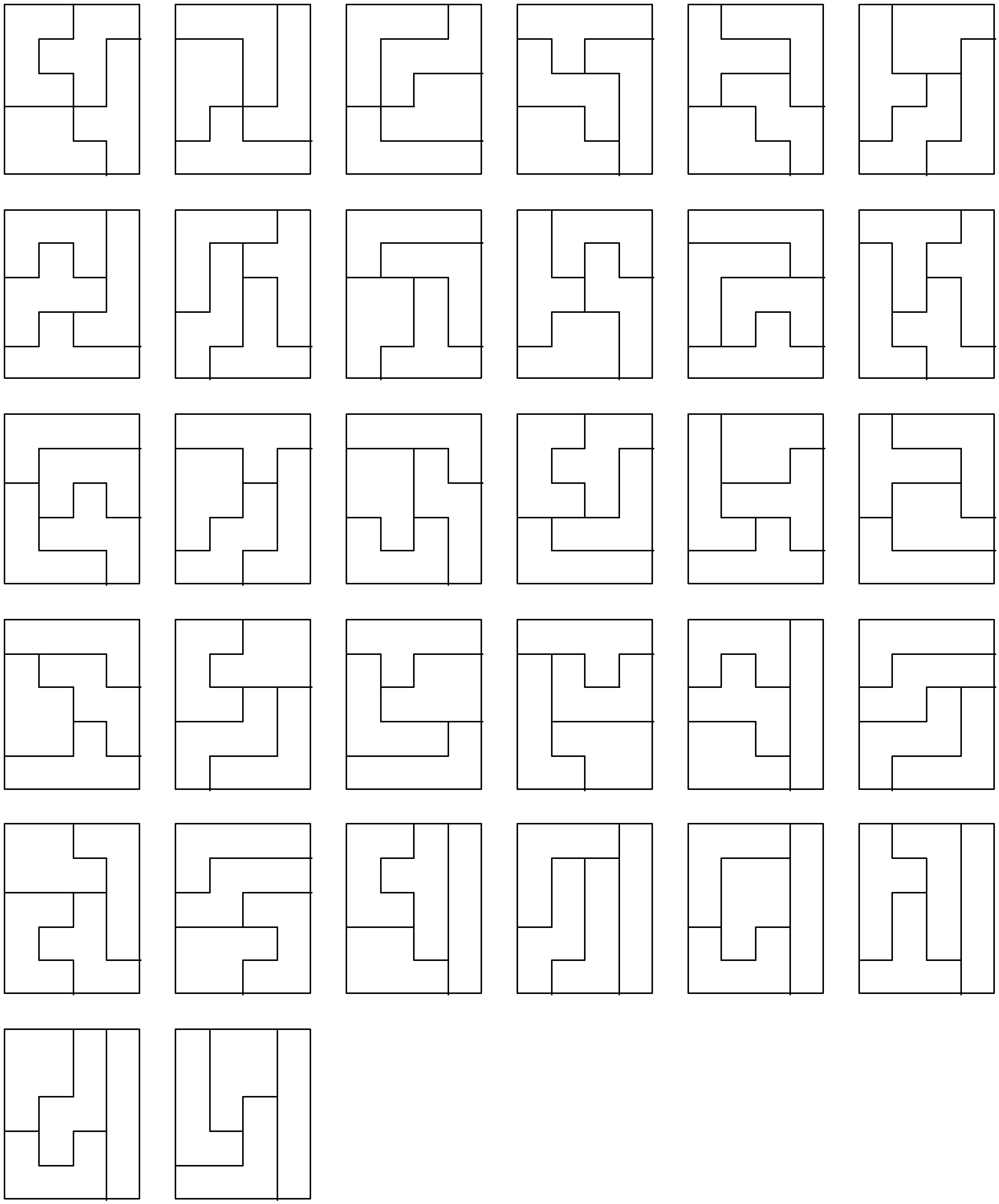
3 x 5

4 x 5

3 x 5 Puzzle Solutions

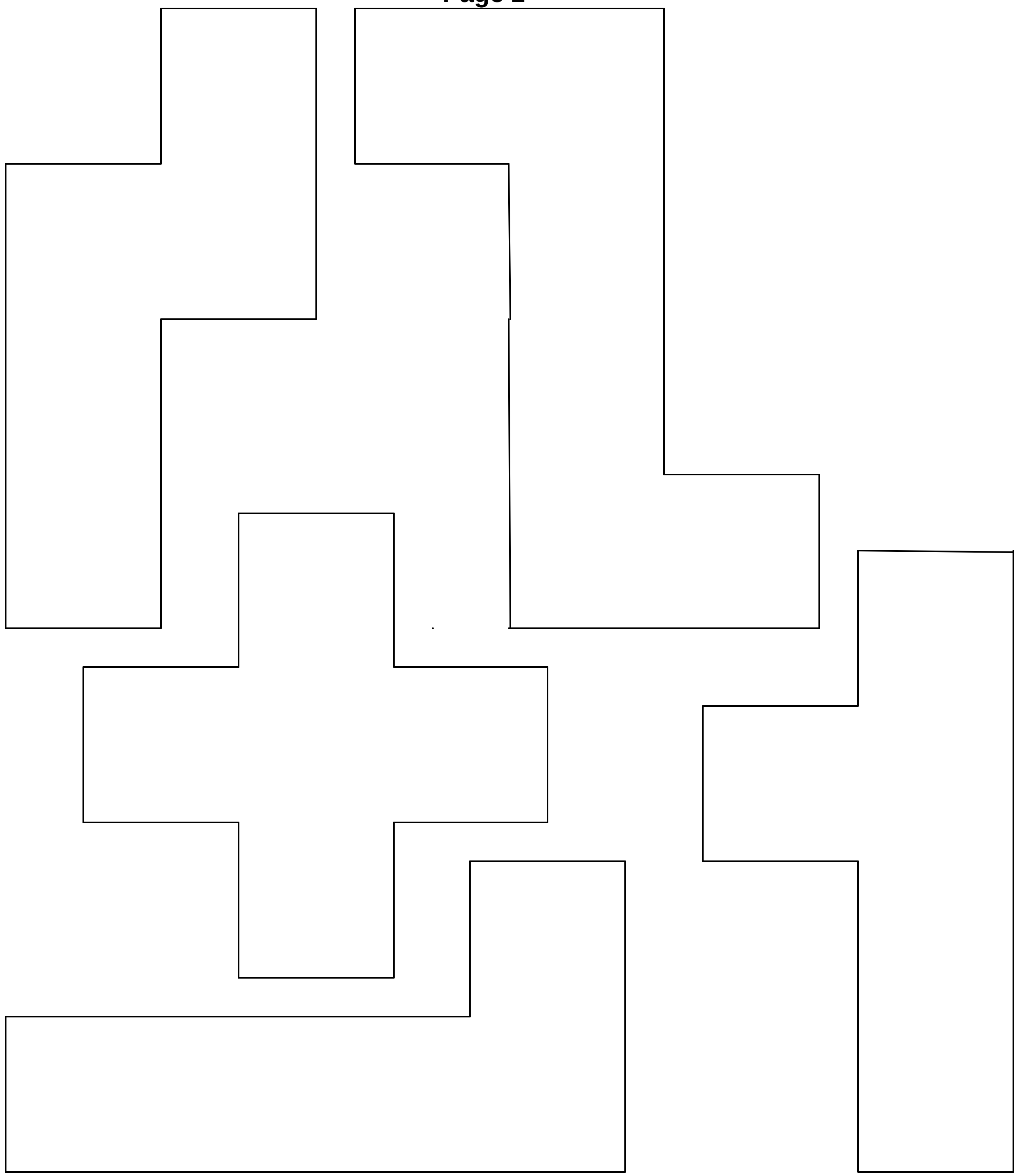


4 x 5 Puzzle Solutions



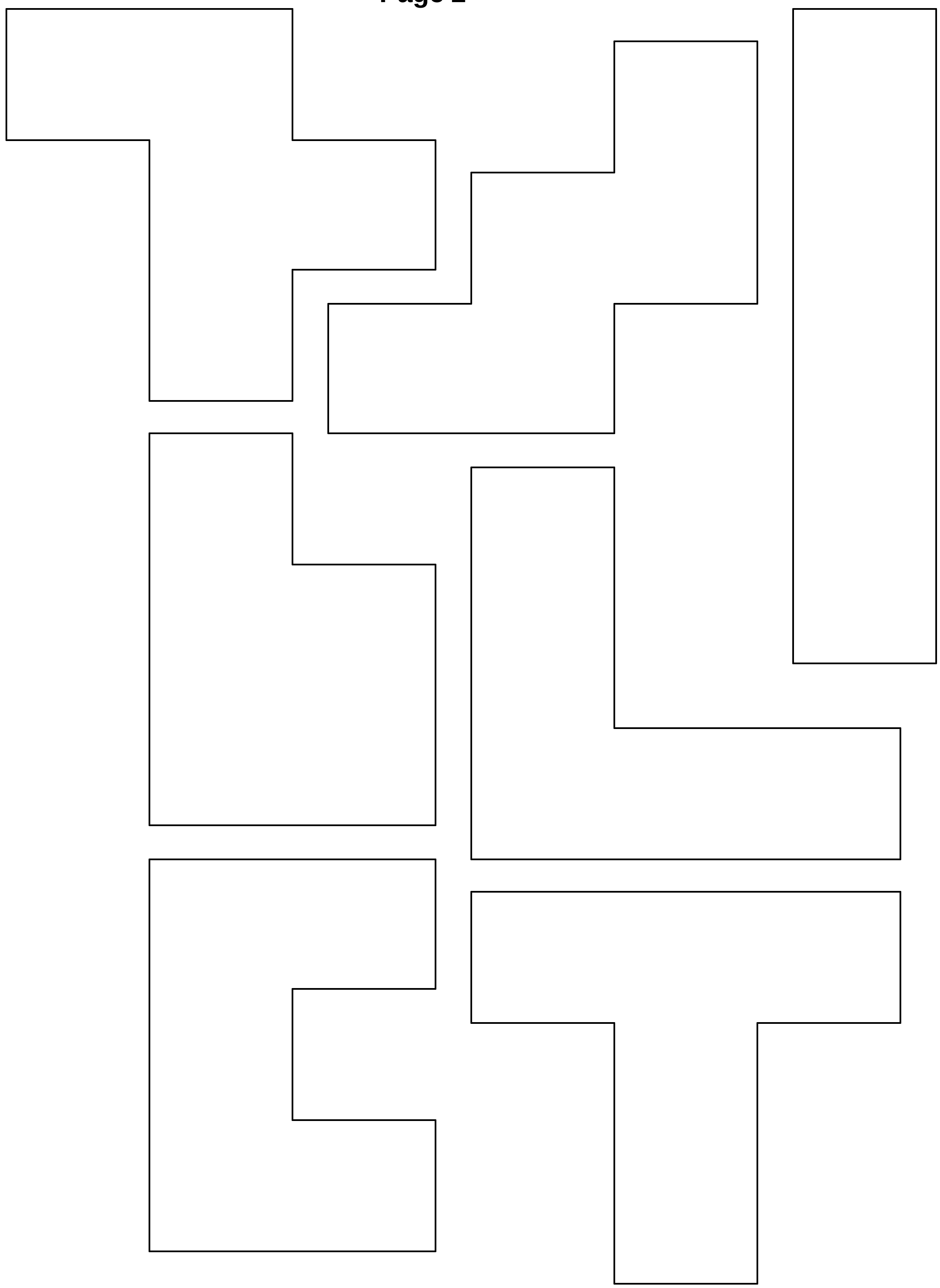
Patterns for Pentominoes

Page 1



Patterns for Pentominoes

Page 2



Activity: Hexominoes

Format: Large Group/Small Group

Objectives: Participants will use spatial visualization to build "hexominoes" with six tiles, testing for figures that are flips (reflections) and turns (rotations) of other figures.

Related SOL: K.11, K.12, K.13, 1.16, 2.22, 3.18, 3.20, 4.17, 5.15

Materials: Tiles, scissors, grid paper, Hexominoes Activity Sheets 1, 2, and 3,, and envelopes or plastic baggies

Time Required: Approximately 15 minutes

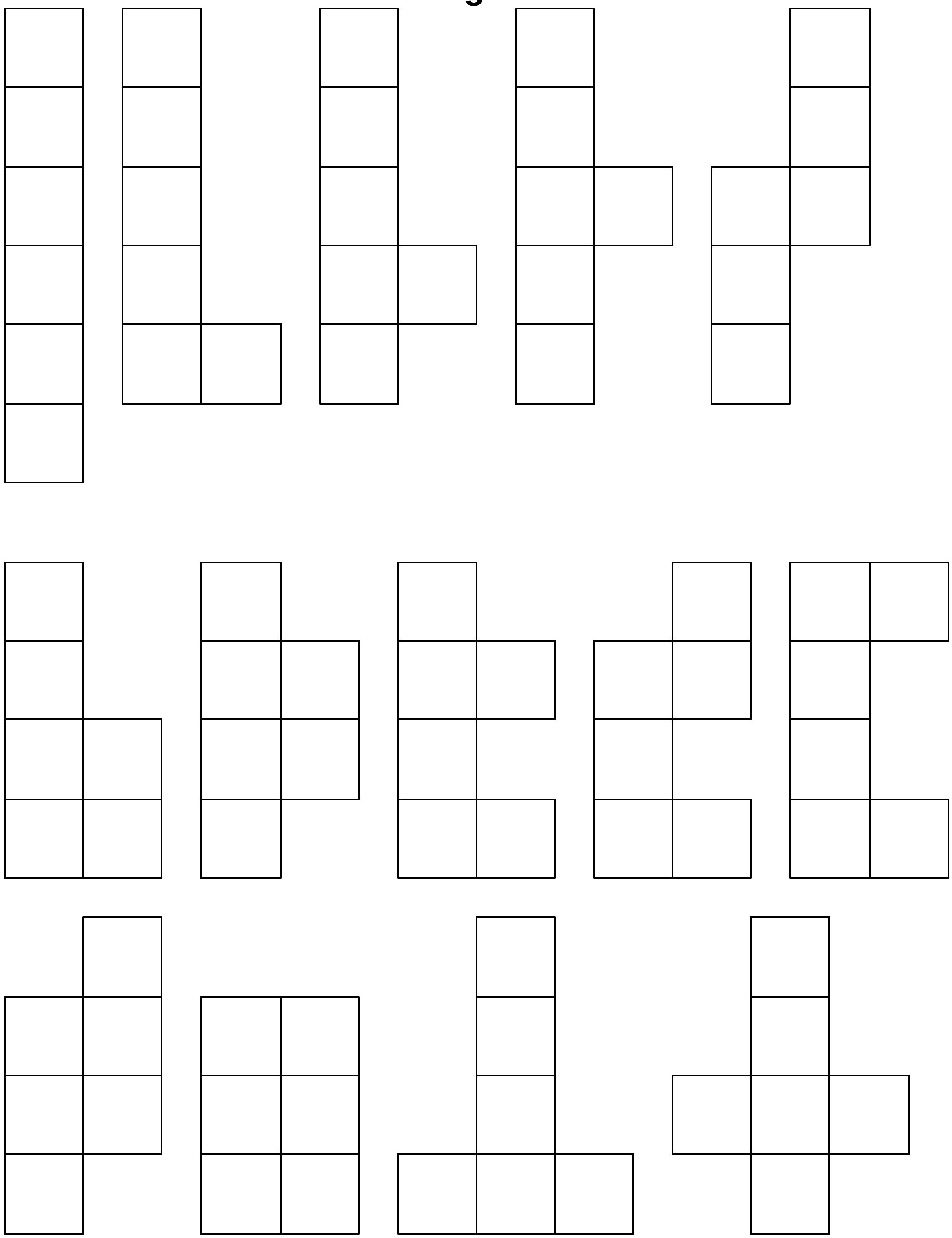
Directions:

1) Distribute the materials and ask the participants to:
   a) On their own or working with a partner, use six color tiles to find all the possible hexominoes. A **hexomino** is a figure made of six squares, each of which has one side in common with the side of another square.
   b) Record each hexomino on grid paper. When you think you have found all the hexominoes cut them out. Check to make sure that each hexomino is different from all the others.
   c) How many hexominoes did you find? (35) Number them on the back and put them in an envelope. Write your names and the number of hexominoes on the envelope.
   d) Exchange envelopes with another pair. Check that none of their hexominoes are duplicates of one another. Mark any two that you think are exactly the same. Return the envelopes. Check your envelope to see if any duplicates were found.
   e) Decide on a way you can sort your hexominoes.

2) Ask one or two pairs to post their hexominoes in an organized way. Use prompts such as these to promote class discussion:
   - Do you think that you have found all possible hexominoes? Why or why not?
   - In what ways do the hexominoes differ from one another?
   - Did you use a strategy to find new figures? If so, what? Note: one strategy would be to start with each of the pentominoes and place one additional tile in each location around the perimeter of each pentomino to generate all of the hexominoes.
   - Did you find any patterns while making your hexominoes? Did you use these patterns when sorting them? If so, how?
   - Did sorting your hexominoes help you find others that were missing? If so, explain.

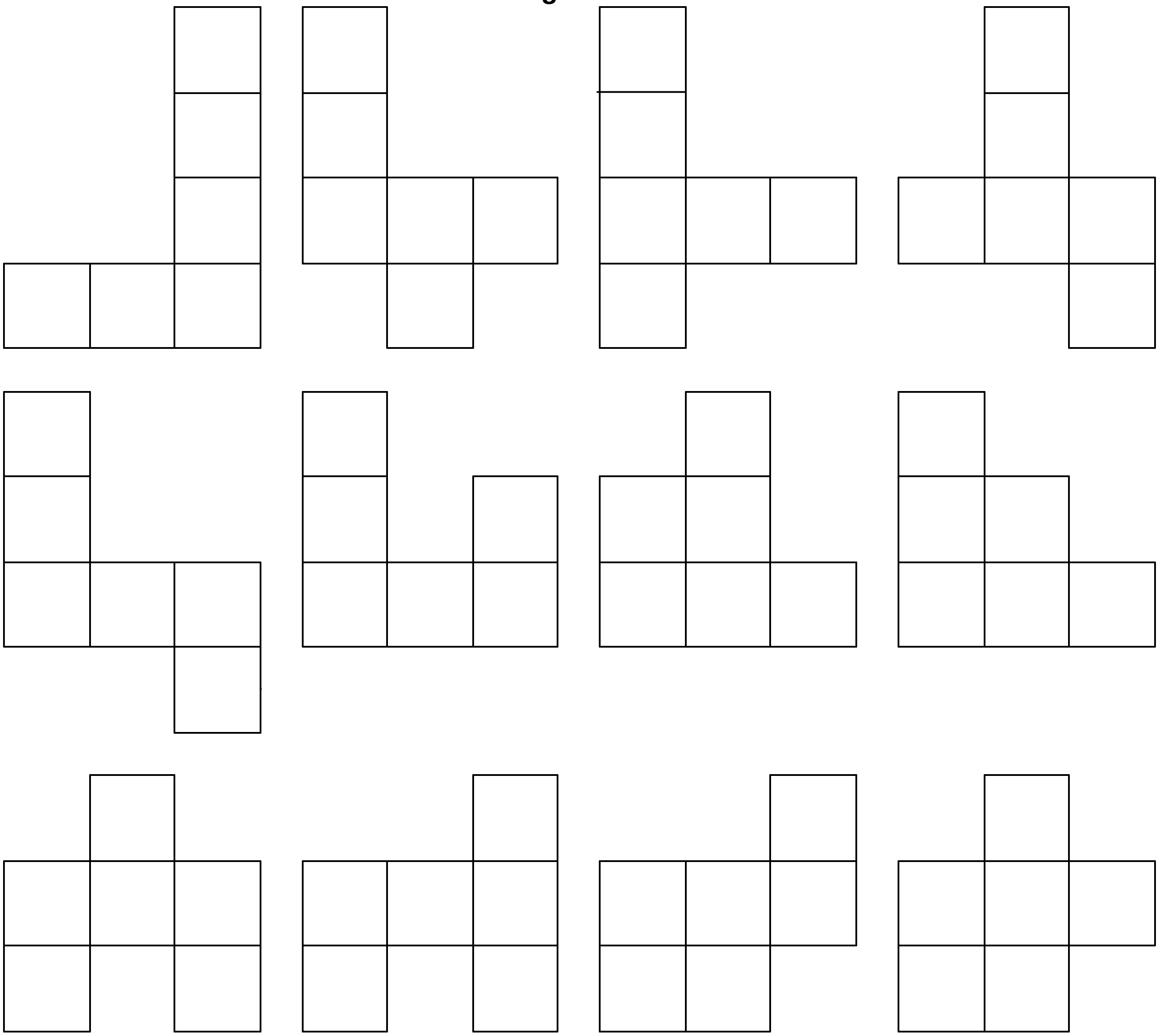
Hexominoes

Page 1



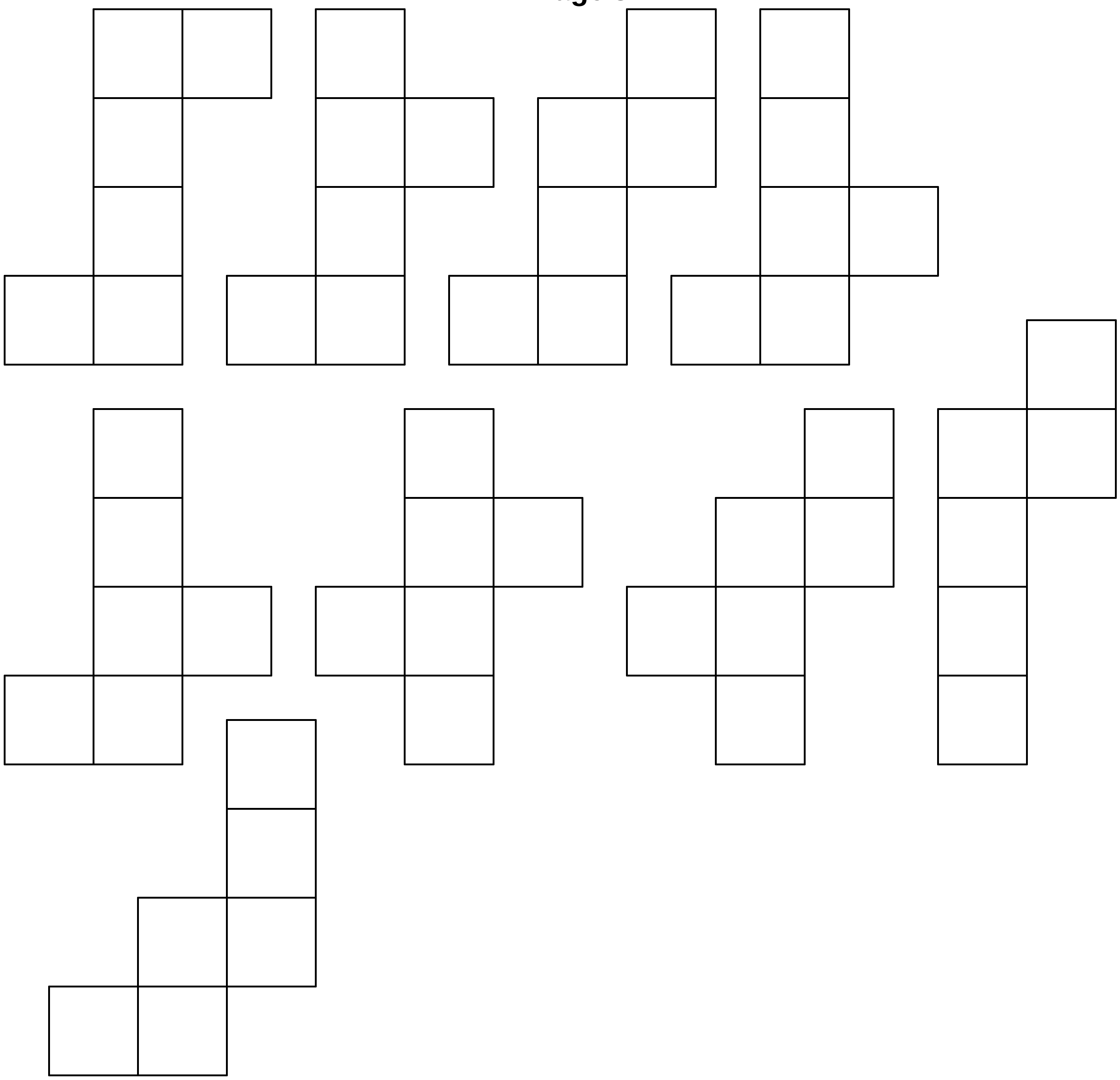
Hexominoes

Page 2



Hexominoes

Page 3



Activity: Perimeters with Hexominoes

Format: Large Group/Small Group

Objectives: Participants will sort "hexominoes" according to their perimeters and look for patterns in their collected data.

Related SOL: K.11, K.12, K.13, 1.16, 2.20, 3.18, 3.20, 4.13, 5.10

Materials: Hexominoes from previous activity, Perimeter of Hexominoes Activity Sheet

Time Required: Approximately 15 minutes

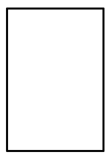
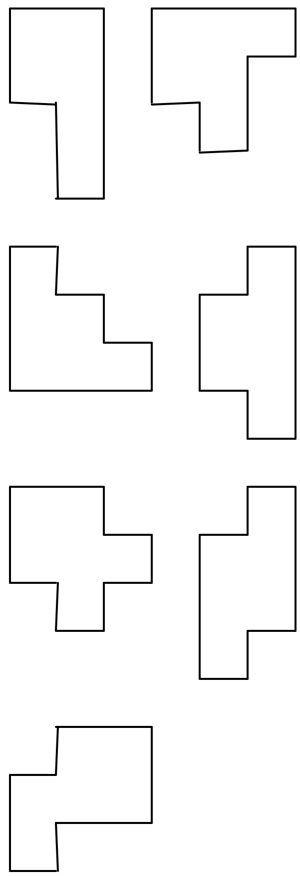
Directions:

1) Ask the participants how they would determine the perimeter of each hexomino. Have them sort all of the hexominoes according to their perimeter and then look for patterns in the collected data.

   Example Patterns:
   - there are no odd numbered perimeters
   - all hexominoes with perimeter 12 include a large square made up of four small squares
   - all hexominoes with perimeter 12 include a rectangle made up of three squares
   - all hexominoes with perimeter 14 include a rectangle made up of two squares
   - none of the hexominoes with perimeter 14 include a large square made up of four small squares

2) Ask the participants to determine the area of each hexomino. Discuss how two figures can have the same area but different perimeters.

Perimeters of Hexominoes

10	11	12	13	14
				**All Others**

Topic: Perimeter and Area, Part II

Description: Participants will explore the relationship between area and perimeter by creating various figures with the same perimeter and different areas or the same area and different perimeter using both one-inch cubes and geoboards. They will also find the areas of rectangles and triangles by counting squares covered by the figures on a geoboard and then develop formulas for area.

Related SOL: 4.13, 5.8, 5.10

Activity: The Perimeter Is 24 Inches. What Is The Area?

Format: Small Group

Objective: Participants will differentiate between perimeter and area, and determine the perimeter and area of various rectangles.

Related SOL: 4.13, 5.8, 5.10

Materials: 24-inch paper strip (collar), 38 one-inch cubes, One-Inch Grid Paper, One-Half-Inch Grid Paper

Time Required: Approximately 10 minutes

Directions:

1) Organize the participants into teams of two or three.

2) Ask the teams to find out how many different rectangular arrays they can make that have a perimeter of 24 inches. Have them make the arrays with the cubes and have them check the perimeter with a 24-inch collar. What is the area of each?

3) Once they find a rectangular array, have them draw its representation on the grid paper and write the perimeter and area for each array.

One-Inch Grid Paper

One-Half Inch Grid Paper

Activity: The Area Is 24 Square Inches. What Is The Perimeter?

<u>Format</u>: Small Group

<u>Objective</u>: Participants will differentiate between the perimeter and area, and will determine the perimeter and area of various rectangles.

<u>Related SOL</u>: 4.13, 5.8, 5.10

<u>Materials</u>: 24 one-inch cubes, One-Inch Grid Paper, One-Half-Inch Grid Paper

<u>Time required</u>: Approximately 10 minutes

<u>Directions</u>:

1) Organize the participants into teams of two or three.

2) Ask the participants to find out how many different rectangular arrays they can make that have an area of 24 square inches. What is the perimeter of each?

3) Once they find a rectangular array of 24 square inches, have them draw its representation on the grid paper and write the perimeter and area for each array.

Activity: Change The Area

Format: Small Group

Objective: Participants will differentiate between perimeter and area and determine the perimeter and area of various figures without dependence on formulas.

Related SOL: 4.13, 5.8, 5.10

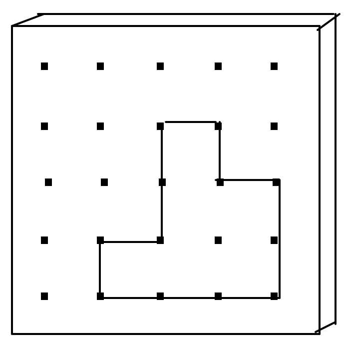
Materials: square geoboards, rubber bands, Geoboard Dot Paper

Time required: Approximately 10 minutes

Directions:

1) Organize the participants into teams of two or three.

2) Have them copy this figure on the geoboard and onto dot paper, labeling its area and perimeter.

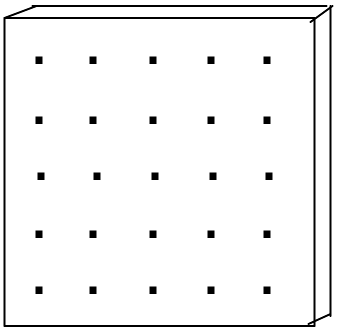


3) Have the participants change the figure to make another figure that has the same area and a larger perimeter, recording it on dot paper with its area and perimeter.

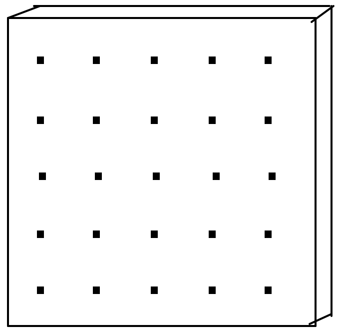
4) Have the participants change the figure to make another figure that has the same area and a smaller perimeter, recording it on dot paper with its area and perimeter.

5) Have the participants make three more figures that have different perimeters but the same area, recording them on dot paper. Discuss the results.

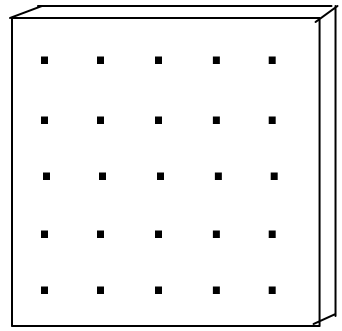
Geoboard Dot Paper



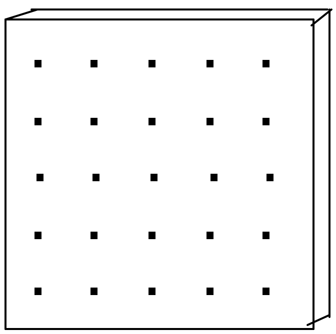
Perimeter: ________
Area: ____________



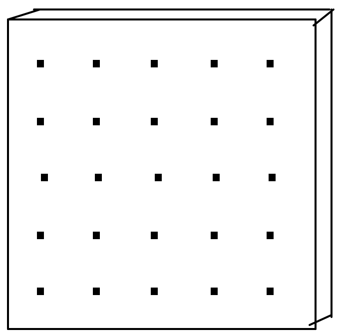
Perimeter: ________
Area: ____________



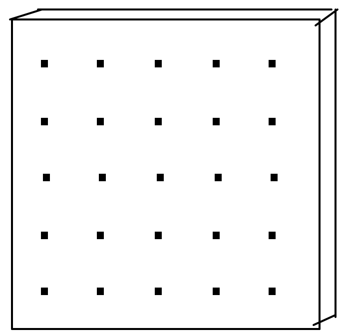
Perimeter: ________
Area: ____________



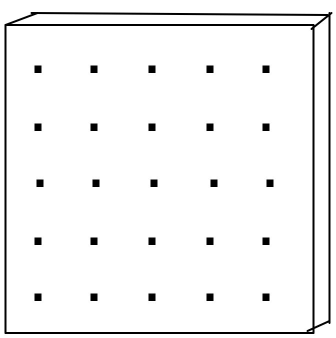
Perimeter: ________
Area: ____________



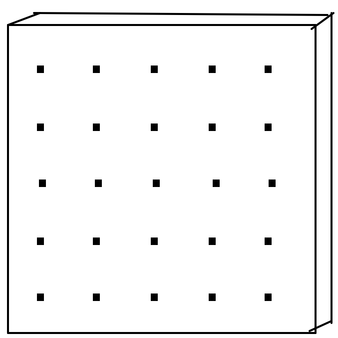
Perimeter: ________
Area: ____________



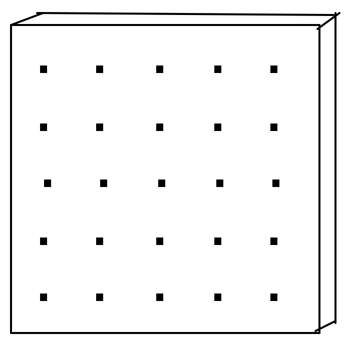
Perimeter: ________
Area: ____________



Perimeter: ________
Area: ____________



Perimeter: ________
Area: ____________



Perimeter: ________
Area: ____________

Topic: Coordinate Geometry

Description: Participants will learn how to label and use ordered pairs of positive numbers on a coordinate grid.

Related SOL: 4.18

Activity: Hurkle

Format: Small Group

Objectives: Participants will review number lines through a simple warm-up exercise. One participant will hide the "Hurkle" on a number line for a partner to find. Participants will use vertical number lines, horizontal number lines, and number lines extending from -5 to +5

Related SOL: 4.18

Materials: Hurkle Activity Sheets 1, 2, and 3

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into pairs

2) Have the first participant draw a number line with 0 as the left endpoint and 10 as the right endpoint.

3) The partner should write down a whole number between 0 and 10 where the first participant can't see it. This number represents the location where the "Hurkle" is hiding.

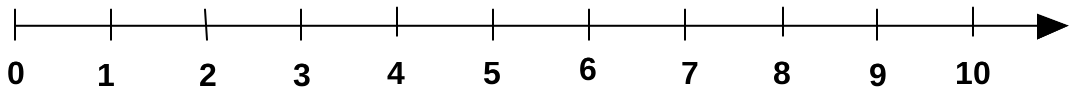
4) The first participant tries to guess the number with the partner responding "higher", "lower", or "perfect".

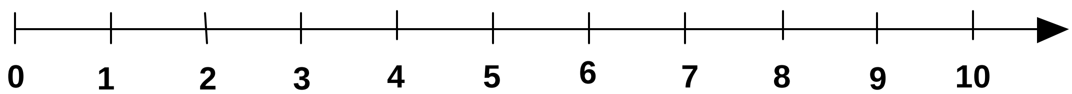
5) Repeat several times, alternating roles, and using vertical number lines, horizontal number lines, and number lines extending from -5 to +5.

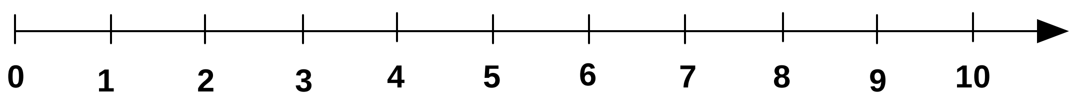
   Note: This warm-up is a non-computer version of the excellent old MECC program *Hurkle* for the Apple II and very early MS-DOS computers.

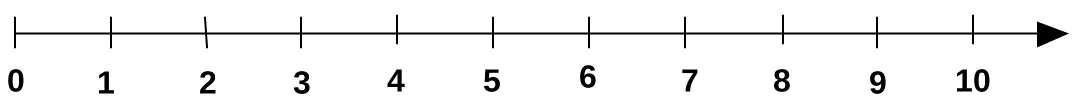
Find the Hidden Hurkle

Page 1









Find the Hidden Hurkle
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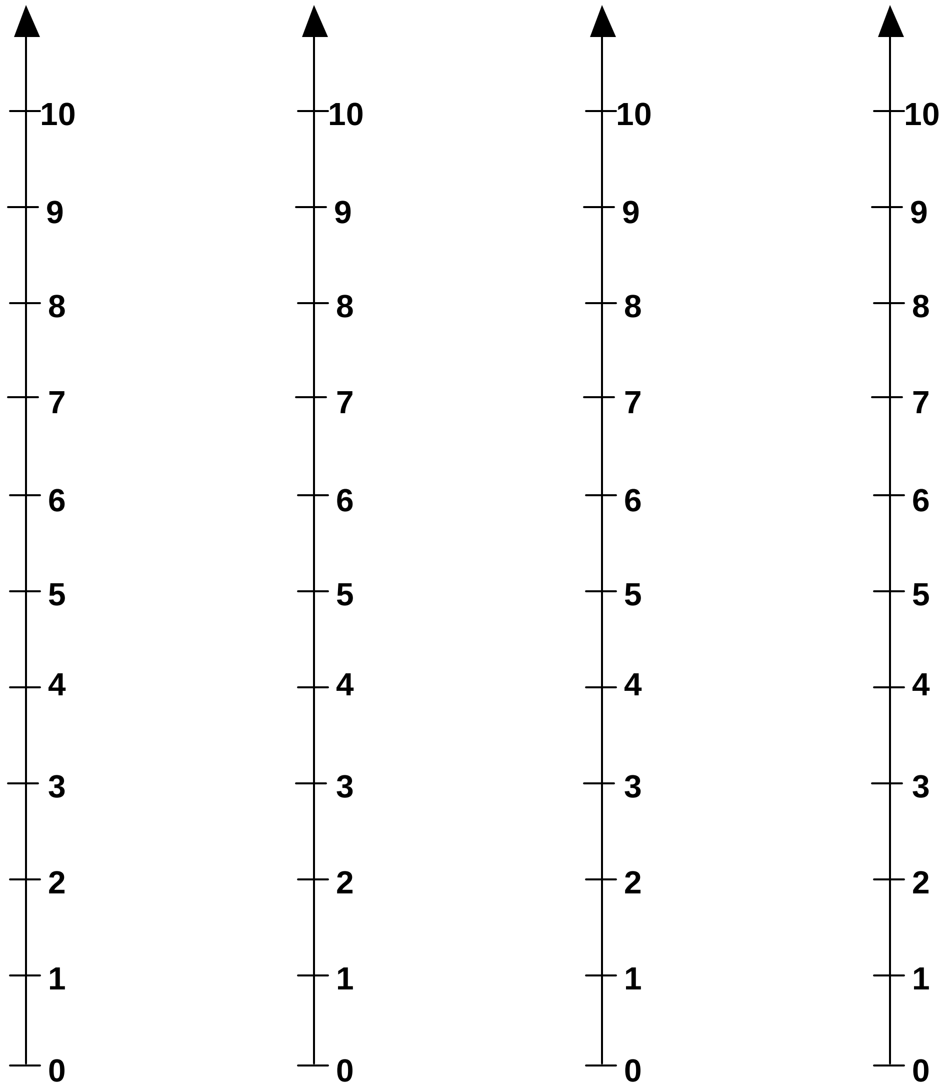

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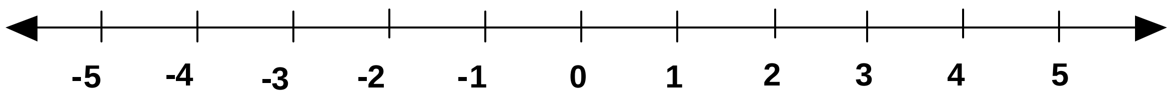

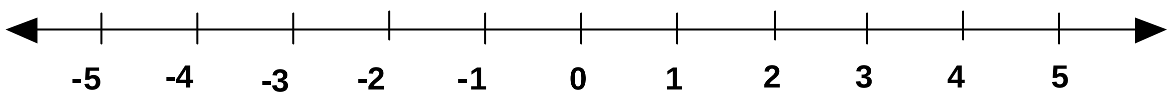

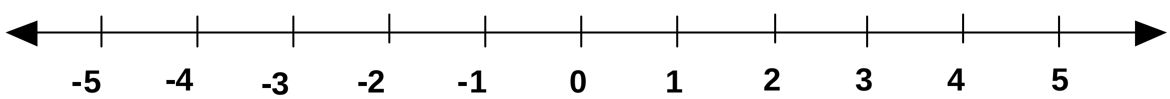

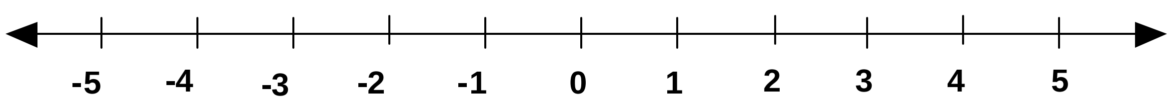

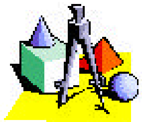

Activity: Two-Dimensional Hurkle

Format: Small Group

Objectives: Participants will learn how to label points on a coordinate grid with an ordered pair of positive numbers and develop problem strategies for finding hidden points.

Related SOL: 4.18

Materials: Find the Hidden Hurkle Coordinate Grid Activity Sheet

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into pairs

2) The first participant should write down an ordered pair of numbers, each between 0 and 10 hidden from the partner. This ordered pair represents the location where the "Hurkle" is hiding.

3) The second participant tries to guess the ordered pair with the partner responding "go right", "go left and down", "go up", "perfect", etc.

4) Repeat several times, alternating roles

   Note: This activity is a further adaptation of the MECC program *Hurkle* for the Apple II and very early MS-DOS computers.

Find the Hidden Hurkle
Coordinate Grid